# INDIAN INSTITUTE OF TECHNOLOGY JODHPUR
## Mathematics-II, Jan-May 2014
## Tutorial Sheet 2 (16 Jan'14)

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1. Reduce the following matrices in to row reduced echelon form by applying elementary row operations.

   (a) $\begin{pmatrix} 0 & 0 & -1 & 2 & 3 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 3 & 2 & 4 & 1 \end{pmatrix}$ $(b)$ $\begin{pmatrix} 1 & -2 & 0 & 2 \\ 2 & -3 & -1 & 5 \\ 1 & 3 & 2 & 5 \\ 1 & 1 & 0 & 2 \end{pmatrix}$

2. Find the rank of the following matrices.

   (i) $\begin{pmatrix} 1 & -1 & 2 & 1 \\ 3 & 2 & 0 & 1 \\ 4 & 1 & 2 & 2 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{pmatrix}$

3. Examine, whether the following matrices are row equivalent or not?

   (a) $\begin{pmatrix} 1 & 0 & 2 \\ 3 & -1 & 1 \\ 5 & -1 & 5 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 10 \\ 2 & 0 & 4 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 2 & 2 \end{pmatrix}$, $\begin{pmatrix} 0 & 3 & -1 \\ 2 & 2 & 5 \end{pmatrix}$

4. Find all values of $\alpha$ for which the equations

   $$\begin{aligned} x + y &= \alpha \\ 3x - \alpha y &= 2 \end{aligned}$$

   have a solution.

5. Find all values of $\alpha$ for which the following equations have (i) a unique solution (ii) no solution and (iii) infinitely many solutions.

   $$\begin{aligned} x_1 + x_2 + x_3 &= \alpha \\ \alpha x_1 + x_2 + 2x_3 &= 2 \\ x_1 + \alpha x_2 + x_3 &= 4. \end{aligned}$$

6. Show that the system

$$\begin{aligned} x_1 - 2x_2 + x_3 + 2x_4 &= 1 \\ x_1 + x_2 - x_3 + x_4 &= 2 \\ x_1 + 7x_2 - 5x_3 - x_4 &= 3 \end{aligned}$$

has no solution.

7. Consider the system of equaitons

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 + 2x_3 &= 1 \\ x_1 - 3x_2 + 4x_3 &= 2. \end{aligned}$$

Does this system have a solution? If so, describe explicitly all solutions.

8. Find the inverse of the matrix by Gauss-Jordan method

(i) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

9. Discuss the existence of the solution for the following system and find them.

$$\begin{aligned} 2x + y + z &= 1 \\ 6x + 2y + z &= -1 \\ -2x + 2y + z &= 7 \end{aligned}$$

10. (H.W.) Solve the following system.

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 1 \\ x_1 + x_2 + 3x_3 + 3x_4 &= 3 \\ x_1 + x_2 + 2x_3 + 3x_4 &= 3 \\ x_1 + 3x_2 + 3x_3 + 3x_4 &= 4 \end{aligned}$$

11. Is the following system consistent? If it is, determine the solution.

$$\begin{aligned} x_1 + x_2 + 2x_3 + 2x_4 + x_5 &= 1 \\ 2x_1 + 2x_2 + 4x_3 + 4x_4 + 3x_5 &= 1 \\ 2x_1 + 2x_2 + 4x_3 + 4x_4 + 2x_5 &= 2 \\ 3x_1 + 5x_2 + 8x_3 + 6x_4 + 5x_5 &= 3 \end{aligned}$$

12. (H.W). Let $A$ be and $n \times n$ real matrix and consider the system $AX = b$. Then prove that the following statements are equivalent.

   (i) The matrix $A$ is invertible.
   (ii) The linear system $AX = b$ has unique solution for every matrix $b$ of order $n \times 1$.
   (iii) The linear system $AX = 0$ has only trivial solution.
   (iv) The matrix $A$ has rank $n$.
   (v) The row-reduced echelon form of $A$ is $I_n$
   (vi) The matrix $A$ is a product of elementary matrices.