CHAOTIC ANALYSIS OF SEMICONDUCTOR LASERS

BY –

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INTRODUCTION:

- What are lasers?

A laser is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. The term "laser" originated as an acronym for Light Amplification by Stimulated Emission of Radiation.

- How can they be chaotic?

Chaos in lasers is related to deterministic chaos in single mode lasers. The onset of deterministic chaos in a dynamical system requires at least a 3-dimensional phase space. We recall that a 3D dynamical system is characterized by 3 coupled first order differential equations as

$$\dot{\vec{x}} = \vec{f}(\vec{x}),$$

with $\vec{x} = (x1, x2, x3)$.

If the system is dissipative, it has attractors, and the sum of the Lyapunov exponents λi of an attractor is negative. This can be satisfied by the following sets of λi signs: (-,-,-); (-,-,0); (-,0,0); (-,0,+). The first set has contraction in all 3 directions, thus yielding a stable equilibrium point attractor. The second set yields a stable limit cycle. The third one corresponds to a torus (quasiperiodic motion with 2 incommensurate basic frequencies). Eventually the fourth one (with the obvious constraint that the positive exponent be smaller than the absolute value of the negative one, in order to satisfy the dissipativity condition) is a "strange" attractor. A positive Liapunov exponent means that an arbitrarily small initial difference between two points on the attractor grows exponentially to a sizable value. This sensitive dependence on the initial conditions has been called "deterministic chaos".

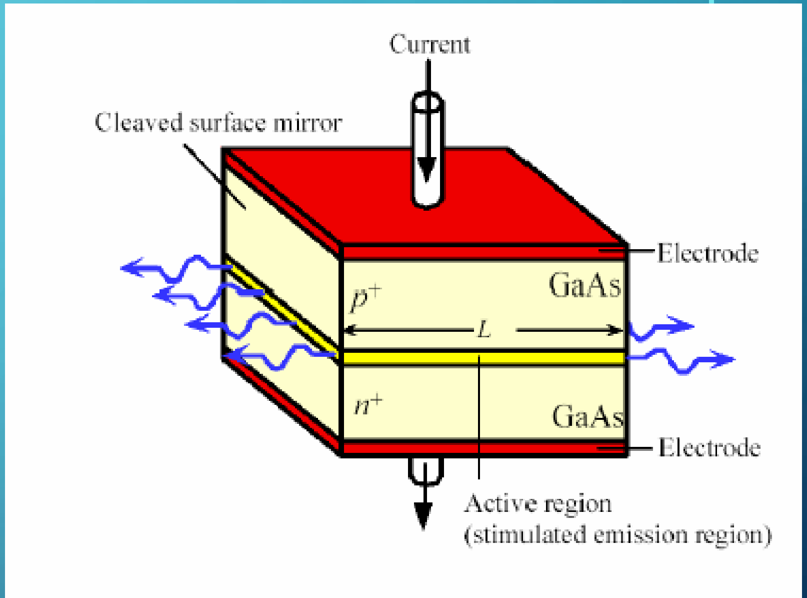
INTRODUCTION..

- How does emission take place in laser?

In order for the laser to emit light population inversion must be present in the system.

Population inversion means that there are more electrons available to make a “down” transition and amplify the light than are available to make an up transition and absorb light. This process occurs as a consequence of pumping in the optical cavity.

Optical cavity is a region composed of two approximately parallel mirrors separated by a defined distance. The semiconducting material used in the laser forms the optical cavity in the case of the semiconductor laser, which is used to produce positive feedback.



- Gain is a measurement that is determined by the length of the optical cavity and the number of reflected passes though the semiconductive material. For each pass through the optical cavity a loss occurs due to the mirrors that is proportional to the gain. When pumping is applied to the semiconductive material, the gain increases for each pass through the optical cavity. Population inversion occurs when the gain reaches a value higher than the loss from reflection.

LASER RATE EQUATIONS:

$$\frac{dE}{dt} = -\alpha_i E + k((N-1) - i\alpha N)E \qquad (1.1)$$

$$\frac{dN}{dt} = J - \gamma N - 2k(N-1)|E|^2 \qquad (1.2)$$

COMPONENT	DESCRIPTION
E	Electric field envelope (Complex amplitude)
N	Carrier density
$-\alpha^i$*E	Loss
$-i\alpha N$	Change to refractive index (dependent on carrier density)
J	Pumping (current)
$-\gamma N$	Loss of carriers from spontaneous recombination
k [(N-1)]E	Gain from stimulated emission
-2k(N-1)abs(E)^2	Consumption of carriers from stimulated emission

STABILITY AND FIXED POINTS

- **Stability of the Intensity System**

To begin, we shall analyze the stability of what is to be called, the Intensity System.

The laser system will be modeled as a system coupled differential equations, as previously studied by [1], which relates the complex amplitude (E) and carrier density (N) as functions of time by:

$$\frac{dE}{dt} = -\alpha_i\, E + k\left((N-1) - i\,\alpha\, N\right)E \qquad (1.1)$$

$$\frac{dN}{dt} = J - \gamma\, N - 2\,k\,(N-1)\,|E|^2 \qquad (1.2)$$

Upon solving the solution takes the form:

$$\alpha_i I = k(N-1) I \Rightarrow I = 0, \; N = \frac{\alpha_i}{k} + 1$$

The former solution suggests that the laser is off, which is not of particular interest. Therefore, to retrieve the corresponding "on" fixed point for the intensity (I) the latter is chosen and used in the zero solution.

In summary, the (on) fixed point of interest is:

$$(I_o, N_o) = \left(\frac{1}{2} \left(\frac{J-\gamma}{\alpha_i} - \frac{\gamma}{k} \right), \; \frac{\alpha_i}{k} + 1 \right)$$

- In order to test the stability of this fixed point a linearization is performed with perturbations variables c and h on Io and No, respectively. Under these stipulations we have that:

$$I_o \to I_o + \chi, \; N_o \to N_o + \eta, \; \ni \chi, \eta \ll 1, \chi \in \mathbb{C}, \; \eta \in \mathbb{R}$$

- On performing the linearization it yields :

$$\frac{d\chi}{dt} = (2(k(N_o - 1) - \alpha_i))\,\chi + (2\,I_o\,k)\,\eta$$

$$\frac{d\eta}{dt} = (1 - N_o)\,\chi - (\gamma + I_o)\,\eta$$

which can be written as

$$\frac{d}{dt}\begin{pmatrix}\chi\\ \eta\end{pmatrix} = \begin{pmatrix} w & x\\ y & z\end{pmatrix}\begin{pmatrix}\chi\\ \eta\end{pmatrix}$$

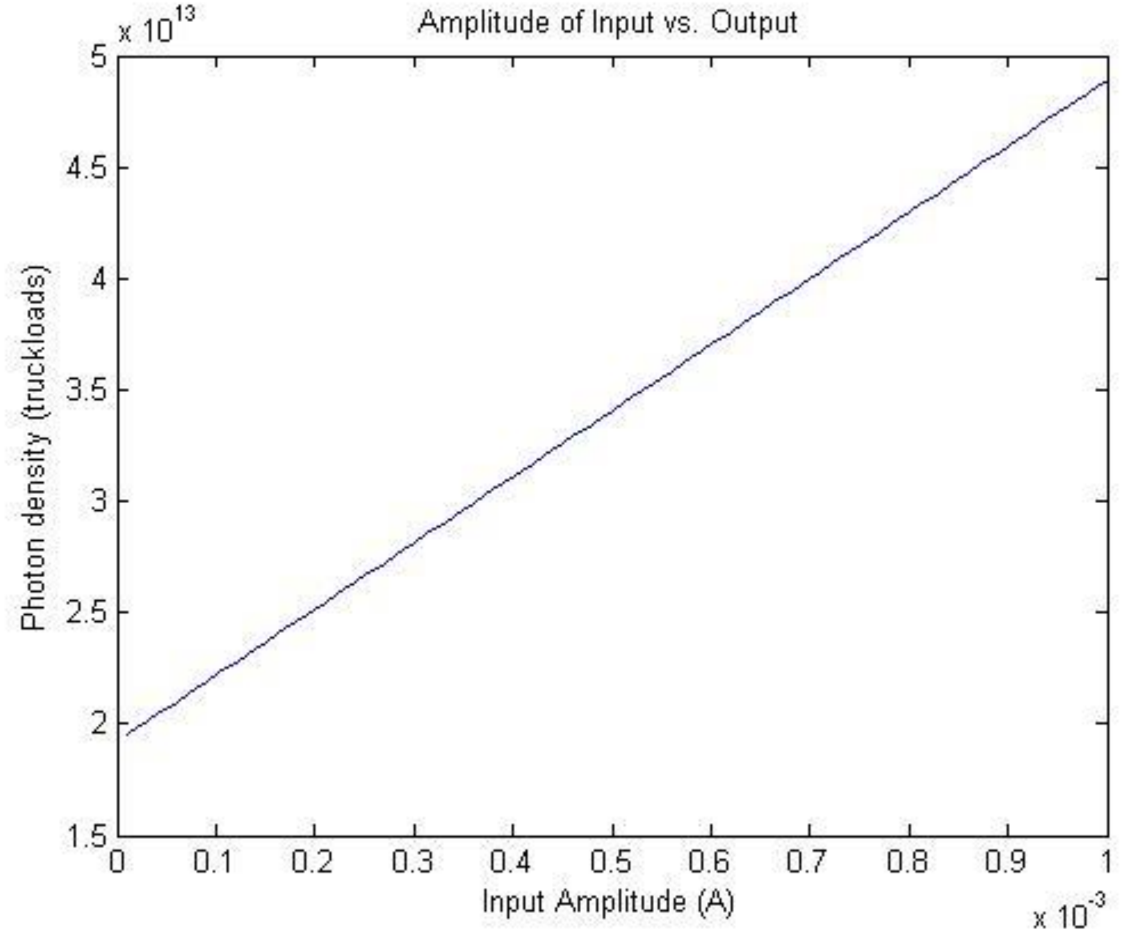
Where $w = 2(-\alpha_i + k(N_o - 1))$,
$x = 2\,I_o\,k$, $y = 1 - N_o$, and $z = -\gamma - I_o$

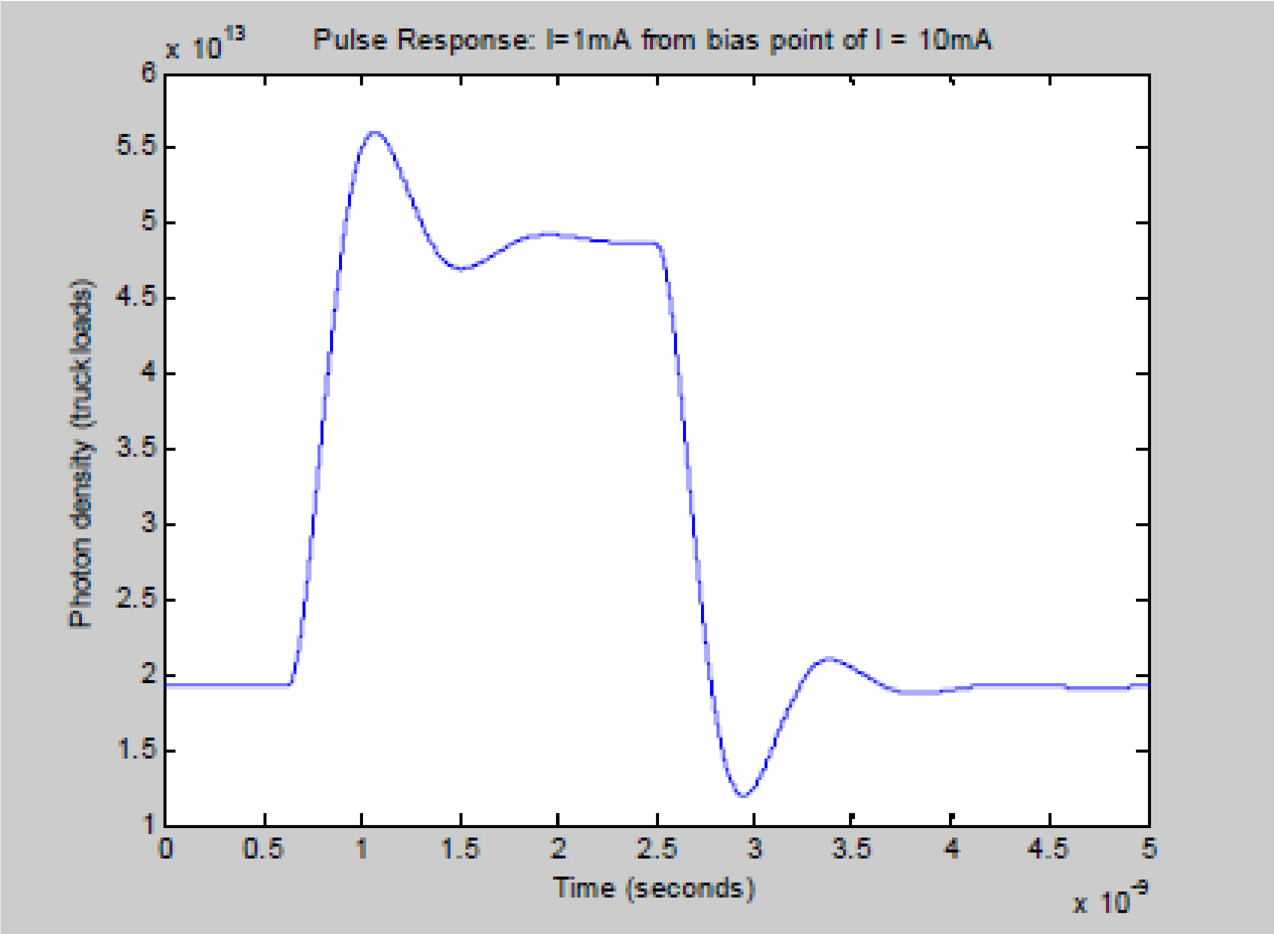
- Upon further calculation and analysis the eigenvalues come as:

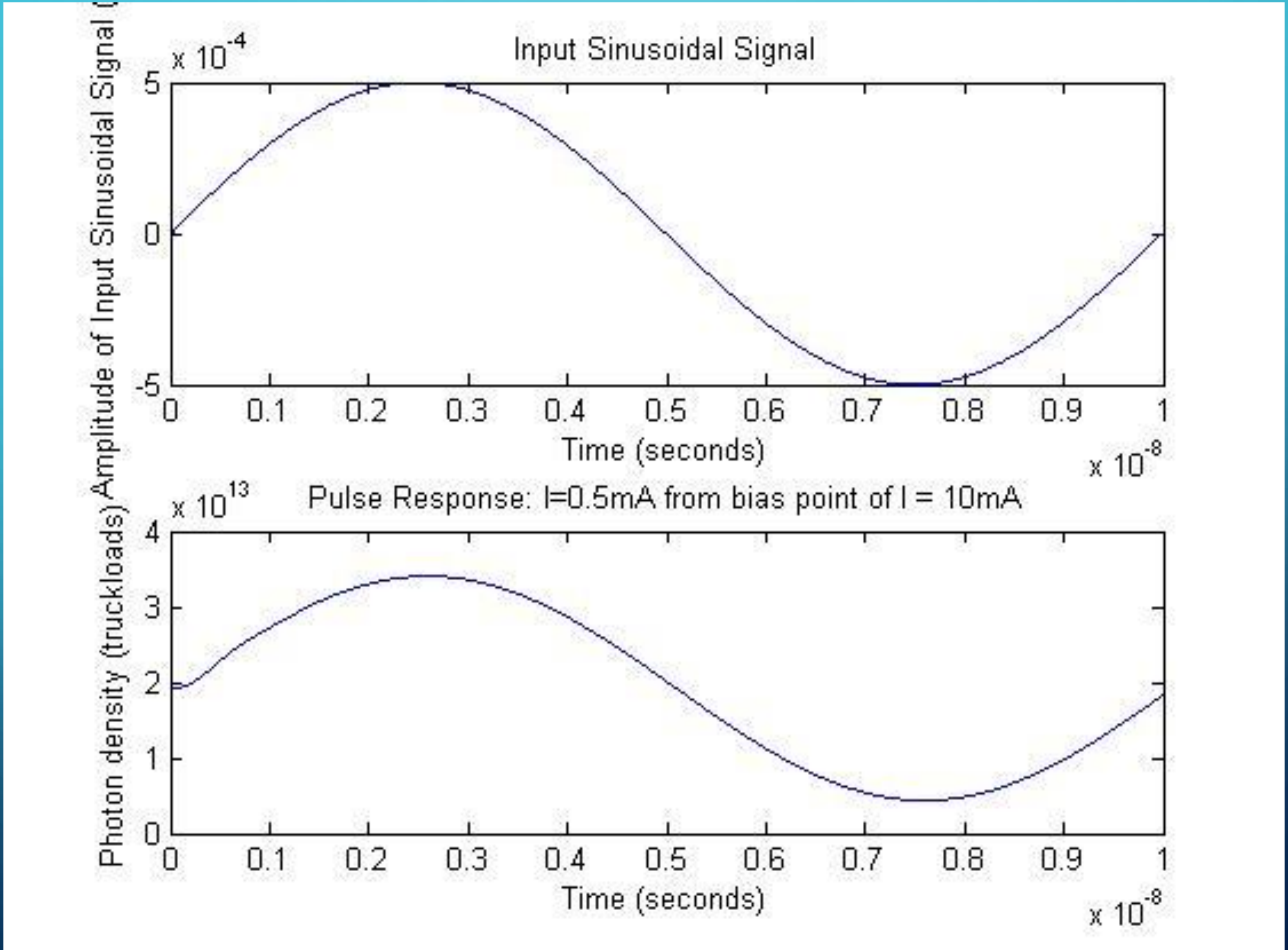
$$\lambda = \frac{-(\gamma + I_o) \pm \sqrt{(\gamma + I_o)^2 + 8(I_o(1 - N_o))}}{2}$$

- Analyzing the above equation shows that the critical points occur at $I_0 = 0$ and $N_0 = 1$. Relating these conditions to previous equations yield that the Intensity System will be stable so long as the current (J) is less than $\gamma(\frac{\alpha_i}{k} + 1)$ and that $\frac{\alpha_i}{k}$ greater than zero.

- The results of the analysis were plotted using matlab.


Amplitude of Input vs. Output
x 10^13
Photon density (truckloads)
Input Amplitude (A)
x 10^-3


Pulse Response: I=1mA from bias point of I = 10mA
x 10^13
Photon density (truck loads)
Time (seconds)
x 10^-9


Input Sinusoidal Signal
x 10^-4
Amplitude of Input Sinusoidal Signal
5
0
-5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time (seconds)
x 10^-8
Pulse Response: I=0.5mA from bias point of I = 10mA
x 10^13
Photon density (truckloads)
4
3
2
1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time (seconds)
x 10^-8

SEMICONDUCTOR LASER SUBJECTED TO A DELAYED OPTICAL FEEDBACK

- Semiconductor lasers with a long external cavity are very sensitive to external signals. The light traveling back and forth in the external cavity takes a long time relative to the internal time scale of the laser, and produces a delayed interaction with a large delay. Because of the large delay, a small amount of optical feedback is enough to produce a variety of instabilities. When the laser is pumped just above threshold, intensity dropouts occur irregularly. This phenomenon, called low-frequency fluctuations.

In these equations, time t is measured in units of the photon lifetime τ_p (τ_p =1ps). T and τ are the carrier lifetime and the external round-trip time, respectively, normalized by τ_p (T≅1000, τ_p ≅1000). ω_0 is the dimensionless frequency of the solitary laser, k is the feedback strength ($0 \leq k << 1$), *P* is the pump current above threshold ($|P| < 1$), and a is the linewidth enhancement factor.

The LK equations are delay differential equations(DDEs), because the right-hand side of Equation on next slide does not only depend on $E(t)$ and $N(t)$ at the present time, but also on $E(t- \tau)$.

LANG KOBAYASHI MODEL

- A minimal description of a single-mode semiconductor laser exposed to weak optical feedback was proposed by Lang and Kobayashi. In dimensionless form, the LK equations consist of two rate equations for the complex electrical field *E*(*t*) and the excess carrier number *N*(*t*). They are given by,

$$\frac{dE}{dt} = (1 + i\alpha)NE + \kappa \exp(-i\omega_0\tau)E(t-\tau),$$

$$T\frac{dN}{dt} = P - N - (1 + 2N)|E|^2.$$

- Hohl and Gavrielides investigated both experimentally and numerically how LFF appears as the result of cascading bifurcations from external cavity modes (ECMs). The ECMs are periodic solutions of Eq. exhibiting a constant intensity, and they sequentially appear as the feedback strength k is increased. In the presence of a small number of ECMs, they observed a series of bifurcations between the destabilization of one ECM and the appearance of the next stable one, which eventually leads to irregular behavior with a broad spectrum and chaotic time traces. They showed how this irregular behavior gradually evolves into LFF for larger values of k and thus more destabilized ECMs.

A detailed numerical bifurcation analysis of the LK equations, using DDE-BIFTOOL. This MATLAB software package calculates steady state and periodic solutions for equations with a finite number of fixed discrete delays. Stability analysis of steady state solutions is achieved through approximating and correcting the rightmost characteristic roots.

In order to study external cavity modes, which are singlefrequency periodic solutions of Eq., and their bifurcations, we transform the original autonomous equations using the substitution:

E(t)=A(t)exp(ibt) …………...2

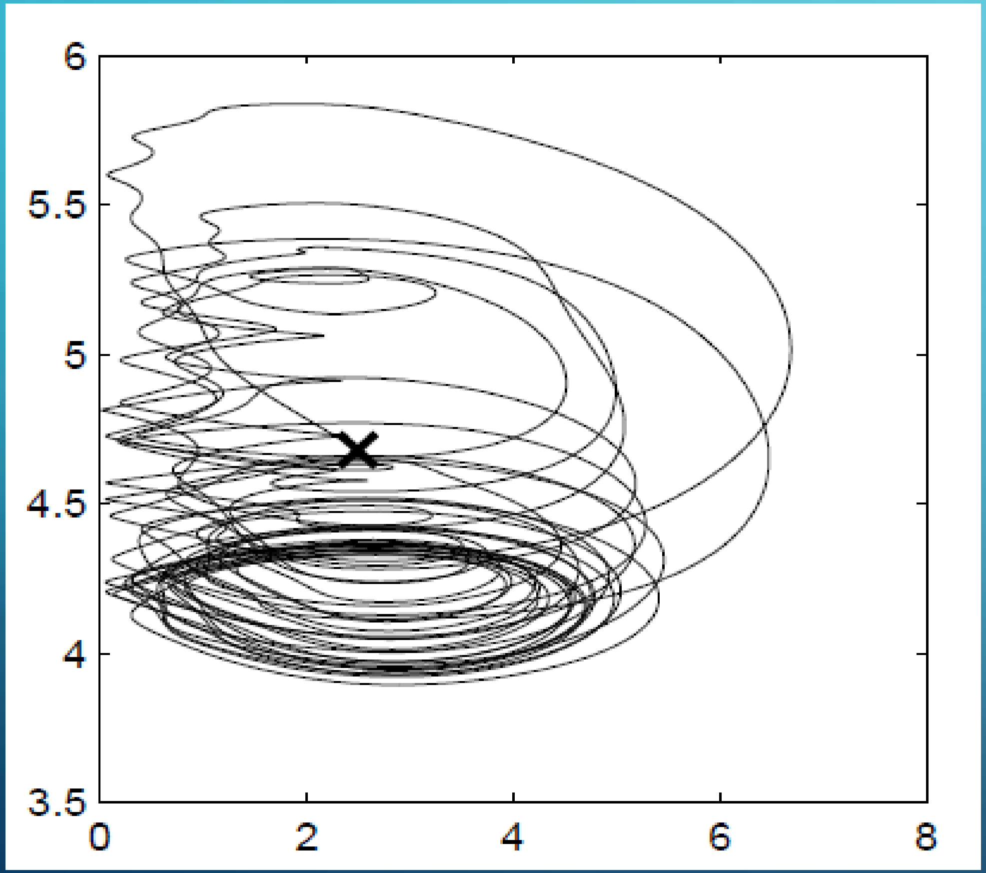
We get,

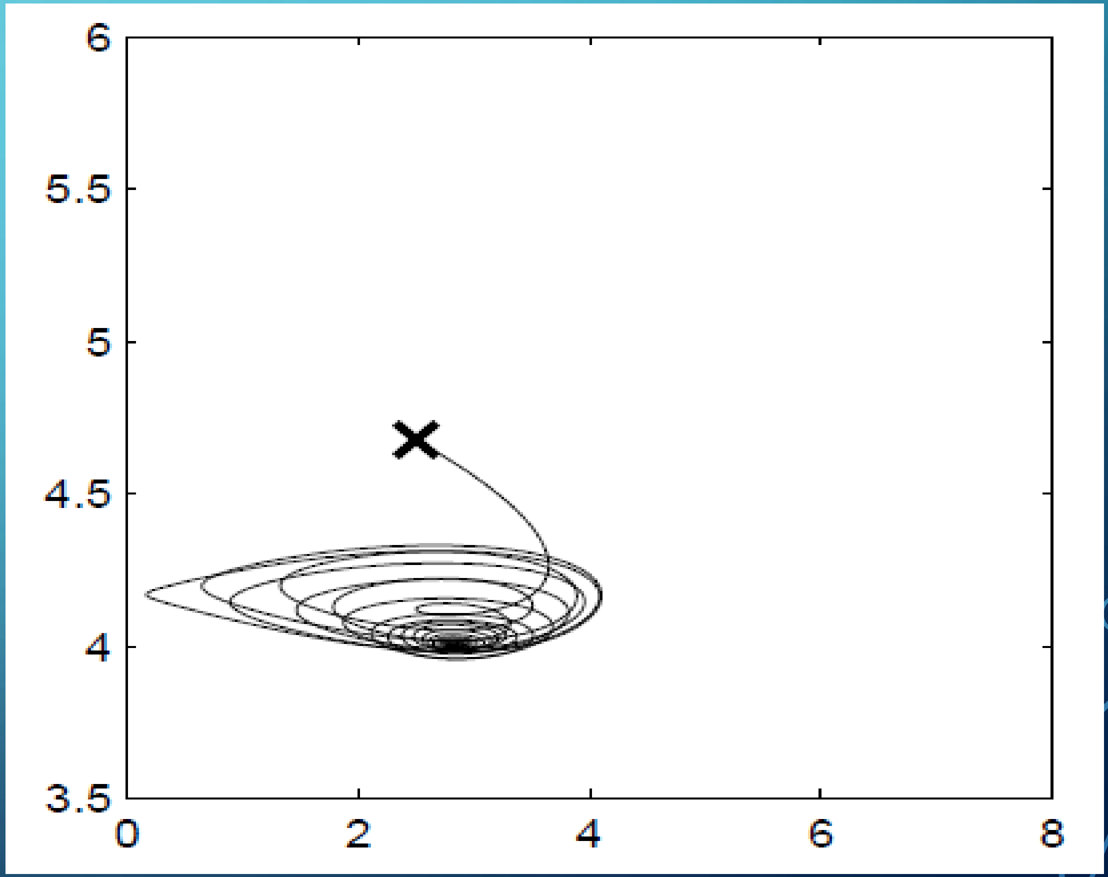
$$\frac{dA}{dt}=(1+i\alpha)NA-ibA+\kappa\exp[-i(\omega_0\tau+b)]A(t-\tau),$$

$$T\frac{dN}{dt}=P-N-(1+2N)|A|^2. \qquad (3)$$

We now have two equations one complex and one real. In the complex variable *A*(*t*) and the real variable *N*(*t*), together with the unknown real parameter *b*. This form has the advantage that the ECMs, which are periodic solutions of Eq.

- Now using the DDE BIFTOOL we plotted two branches of the 1D unstable manifold showing bistability between a fixed point and a chaotic attractor in the COF laser.





CONCLUSIONS

- During this project we studied the Lang-Kobayashi equations for semiconductor lasers subject to optical feedback. Our study is based on the application of the numerical techniques implemented in the package DDE-BIFTOOL for stability and bifurcation analyses of delay differential equations.
- This analysis reveals that increasing the linewidth enhancement factor progressively changes the stability of the bridge, but α must be high enough (α=6) for rupture.

REFERENCES

- Stability and rupture of bifurcation bridges in semiconductor lasers subject to optical feedback B. Haegeman,* K. Engelborghs, and D. Roose *Department of Computer Science, K.U. Leuven, Celestijnenlaan 200A, 3001 Heverlee, Belgium*
- Analysis of Chaotic Semiconductor Laser Diodes J. P. Toomey and D. M. Kane Department of Physics Macquarie University, Sydney, Australia
- Simulation and Analysis of Single Mode Semiconductor Laser Guangjian Feng and Xavier Fernando Electrical and Computer Engineering Ryerson University
- DDE-BIFTOOL v. 2.00: a Matlab package for bifurcation analysis of delay differential equations. K. Engelborghs, T. Luzyanima, G. Samaey Report TW 330, October 2001.

- Optical communication using chaos based transmission scheme, G Kandiban & V Balachandran, Dept. Of Physics Thanthai Hans Roever College, Perambalur Dept. of Physics, A A Govt. Arts College, Muisiri, Tiruchirappali
- Chaotic Effects in FLARED LASERS:A Numerical Analysis, Guy Levy and Amos A. Hardy, *Senior Member, IEEE*
- Chaos and Locking in a Semiconductor Lase Due to External Injection, V. Annovazzi-Lodi, Member IEEE, S. Donati, Member IEEE, and M.Manna
- Analysis and Characterization of the Hyperchaos Generated by a Semiconductor Laser Subject to Delay feedback loop
- Chapter 5 Bifurcation analysis of lasers with delay Bernd Krauskopf University of Bristol Chapter for a book edited by Deb Kane and Alan Shore

THANK YOU