REVIEW OF "CIRCUIT IMPLEMENTATION OF SYNCHRONIZED CHAOS WITH APPLICATION TO COMMUNICATION"

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INTRODUCTION:

- In this Project, we focus on the synchronizing properties of the Lorentz system, the implementation of the Lorentz system as an analog circuit, and the potential for utilizing the circuit for various communication applications.
- It Should be noted that the applications indicated are very preliminary and presented to suggest and illustrate possible directions.

WHAT IS SELF SYNCHRONIZING PROPERTY ?

- A chaotic system is said to be self synchronizing if it can be decomposed into subsystems: a drive system and a stable response subsystem that synchronize when coupled with a common drive signal.
- The Lorenz system is decomposable into two separate response subsystems that will each synchronize to the drive system when started out from any initial condition

Lorenz equation is given by:

$$\dot{x} = \sigma(y - x),$$
$$\dot{y} = rx - y - xz,$$
$$\dot{z} = xy - bz, \qquad (1)$$

Where σ, r and b are parameters.

It is decomposable into 2 stable subsystems:

$$\dot{x}_1 = \sigma(y - x_1),$$
$$\dot{z}_1 = x_1 y - b z_1, \qquad (2)$$

First stable response system (x1,z1)

$$\dot{y}_2 = rx - y_2 - x z_2,$$
$$\dot{z}_2 = x y_2 - b z_2. \qquad (3)$$

Second stable response system (y2,z2)

- Now the equation (1) can be interpreted as the drive system since its dynamics are independent of the response subsystem
- Equation (2) and Equation (3) represents the dynamical response systems which are driven by the drive signal y(t) and x(t) respectively.

PROOF OF THE STABILITY OF THE RESPONSE SUBSYSTEMS:

- The eigenvalues of the Jacobian matrix for the *(x1,z1)* subsystem are both negative and thus $|x_1 - x|$ and $|z_1 - z| \to 0$ as $t \to \infty$.
- Also, it can be shown numerically that the lyapunov exponents of the (y2,z2) subsystem are both negative and thus $|y_2 - y|$ and $|z_2 - z| \to 0$ as $t \to \infty$.
- The lorenz system decomposable into two separate response subsystems that will each synchronize to the drive system when started out from any initial condition.

IMPLEMENTATION IN ELECTRONICS CIRCUITS:

- A direct implementation of Eq. (1) with an electronic circuit presents several difficulties. For example, the state variables in Eq. (1) occupy a wide dynamic range with values that exceed reasonable power supply limits.
- However, this difficulty can be eliminated by a simple transformation of variables. Specifically, we define new variables by $u = x/10$, $v = y/10$, and $w = z/20$. With this scaling, the Lorenz equations are transformed to :

$$\dot{u} = \sigma(v - u)\,,$$
$$\dot{v} = ru - v - 20uw\,, \qquad (4)$$
$$\dot{w} = 5uv - bw\,.$$

- This system, which we refer to as the transmitter, can be more easily implemented with an electronic circuit because the state variables all have similar dynamic range and circuit voltages remain well within the range of typical power supply limits.

ANALOG CIRCUIT IMPLEMENTATION OF THE CIRCUIT EQ. (4):

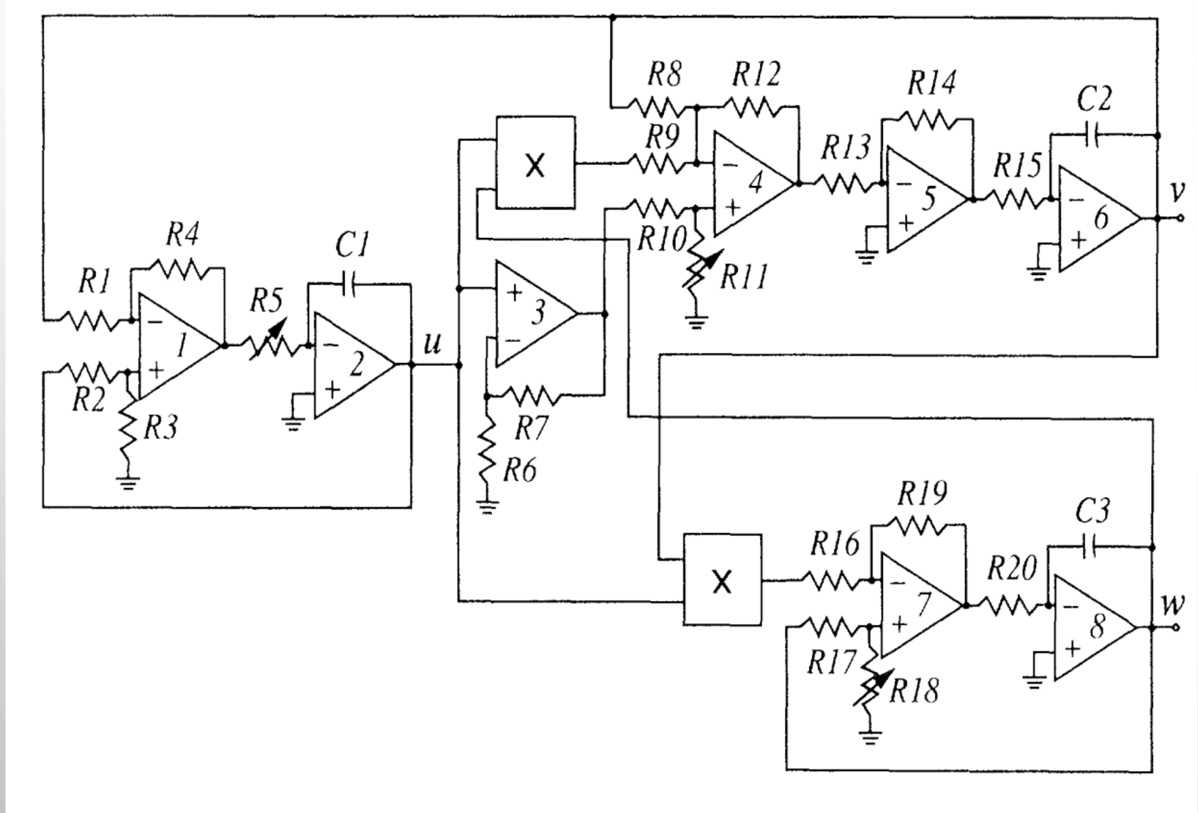


FIG. 1. Lorenz-based chaotic circuit.

- The operational amplifiers (1—8) and associated circuitry perform the operations of addition, subtraction, and integration. Analog multipliers im-plement the nonlinear terms in the circuit equation.
- The coefficients σ, r, and b can be independently varied by adjusting the corresponding resistors R5, R11,and R18. In addition, the circuit time scale can be easily adjusted by changing the values of the three capacitors, C1, C2, and C3, by a common factor. We have chosen component values [Resistors (in KΩ): R1 ,R2, R3, R4,R 6,R7, R13,R14,R16,R17,R19= 100; R5,R10=49.9; R8 =200; R9, R12=10; R11=63.4; R15=40.2; R18=66.5; R20=158;
- Capacitors (pF): C1,C2,C3 = 500; Op-Amps (1-8): LF353 ; Multipliers: AD632AD] which result in the coefficients σ=16, r = 45.6, and b = 4

A set of state equations which govern the dynamical behavior of the circuit is given by :

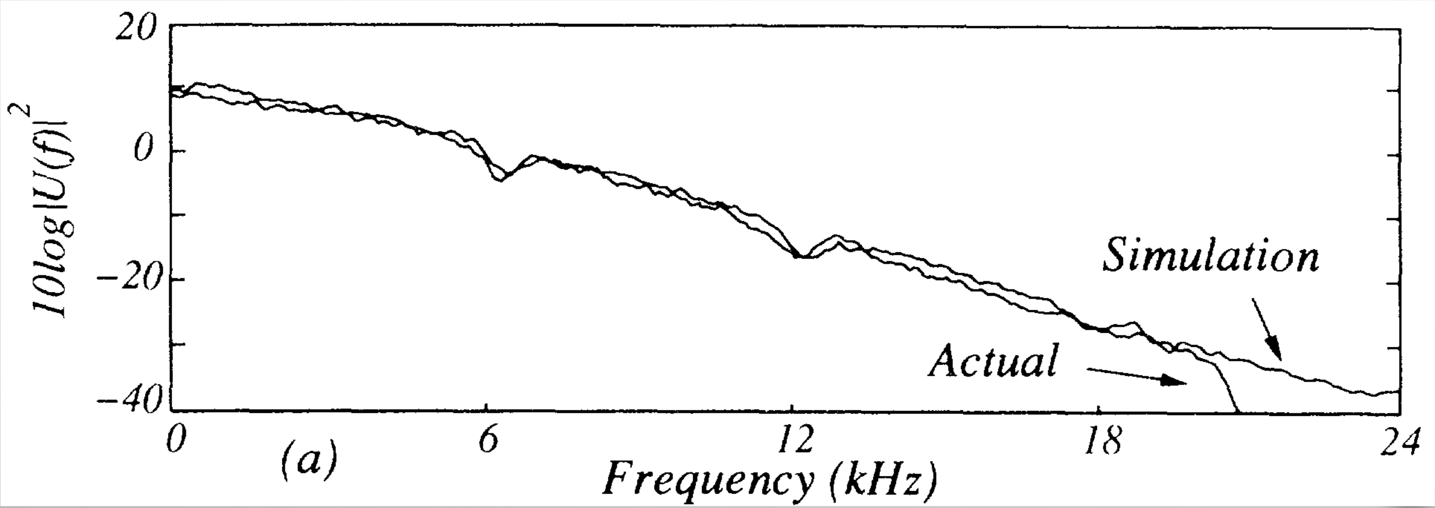
$$\frac{du}{dt} = \frac{1}{R_5C_1}\left[\frac{R_4}{R_1}v - \frac{R_3}{R_2+R_3}\left(1+\frac{R_4}{R_1}\right)u\right]$$

$$\frac{dv}{dt} = \frac{1}{R_{15}C_2}\left[\frac{R_{11}}{R_{10}+R_{11}}\left(1+\frac{R_{12}}{R_8}+\frac{R_{12}}{R_9}\right)\left(1+\frac{R_7}{R_6}\right)u - \frac{R_{12}}{R_8}v + \frac{R_{12}}{R_9}uw\right]$$

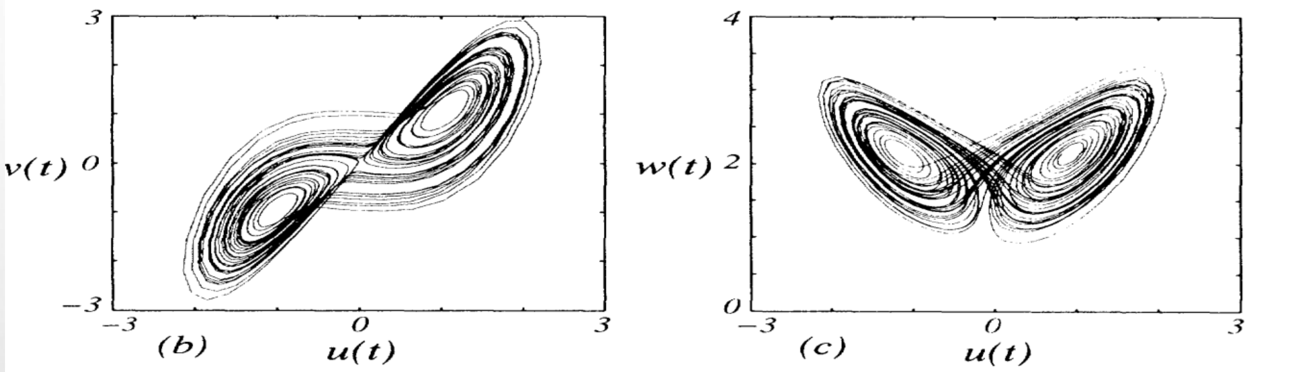
$$\frac{dw}{dt} = \frac{1}{R_{20}C_3}\left[\frac{R_{19}}{R_{16}}uv - \frac{R_{18}}{R_{17}+R_{18}}\left(1+\frac{R_{19}}{R_{16}}\right)w\right]$$

AVERAGE POWER SPECTRUM:

- Figure shows the averaged power spectrum of the circuit wave form u(t). The power spectrum is broadband which is typical of a chaotic signal.
- As we see, the performance of the circuit and the simulation are consistent.



(b) CHAOTIC ATTRACTOR PROJECTED ONTO THE *UV* PLANE; (c) CHAOTIC ATTRACTOR PROJECTED ONTO THE *UW* PLANE.:



- A full-dimensional response system which will synchronize to the chaotic signals at the transmitter (4) is given by:

$$\dot{u}_r = \sigma(v_r - u_r),$$
$$\dot{v}_r = ru - v_r - 20uw_r,$$
$$\dot{w}_r = 5uv_r - bw_r.$$

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- We refer to this system as the receiver in light of some potential communications applications. We denote the transmitter state variables collectively by the vector d $= (u, v, w)$ and the receiver variables by the vector r= $(u_{r,} v_{r,} w_r)$ when convenient.
- By defining the dynamical errors by e = d - r, it is straightforward to show that synchronization in the Lorenz system is a result of stable error dynamics between the transmitter and receiver. Assuming that the transmitter and receiver coefficients are identical, a set of equations which govern the error dynamics are given by:

$$\dot{e}_1 = \sigma(e_2 - e_1) ,$$
$$\dot{e}_2 = -e_2 - 20u(t)e_3 ,$$
$$\dot{e}_3 = 5u(t)e_2 - be_3 .$$

- The error dynamics are globally asymptotically stable at the origin provided that σ,b > 0

USING CHAOS IN COMMUNICATION:

(I) CHAOTIC COMMUNICATION SYSTEM-

1. As one illustration of the potential use of synchronized chaotic systems in communications, we describe a system to transmit and recover binary-valued bit streams.
2. The basic idea is to modulate a transmitter coefficient with the information-bearing wave form and to transmit chaotic drive signal.

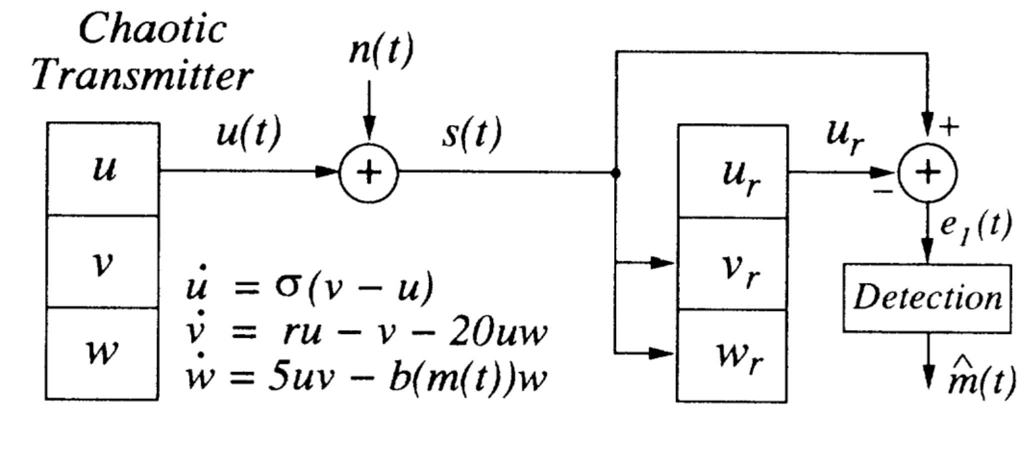

FIG. 3. Chaotic communication system.

3. At the receiver, the coefficient modulation will produce a synchronization error between the received drive signal and the receiver's regenerated drive signal with an error signal amplitude that depends on the modulation. Using the synchronization error the modulation can be detected.

4. The modulation/detection process is illustrated in the figure. In the figure coefficient b of the transmitter equations (4) is modulated by the information-bearing wave form, m(t). For the purpose of demonstrating the technique, we use a square wave for m(t) as illustrated in Fig. 4(a).

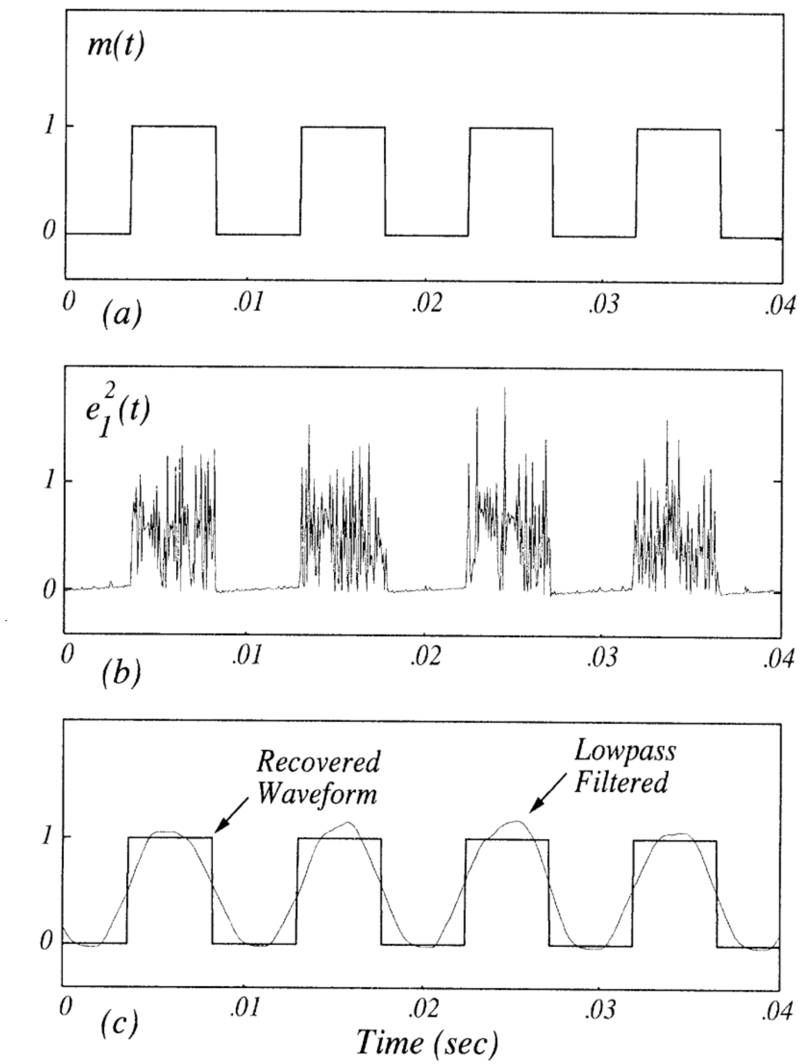

FIG. 4. Circuit data: (a) modulation wave form; (b) synchronization error power; (c) recovered wave form.

5. Figure 4(b) shows the synchronization error power, $e_1^2(t)$, at the output of the receiver circuit, the coefficient modulation produces significant synchronization error during a "1" transmission and very little error during a "0" transmission.

6. Figure 4(c) illustrates that the square-wave modulation can be reliably recovered by low pass filtering the synchronization error power wave form and applying a threshold test.

(II) CHAOTIC SIGNAL MASKING SYSTEM-

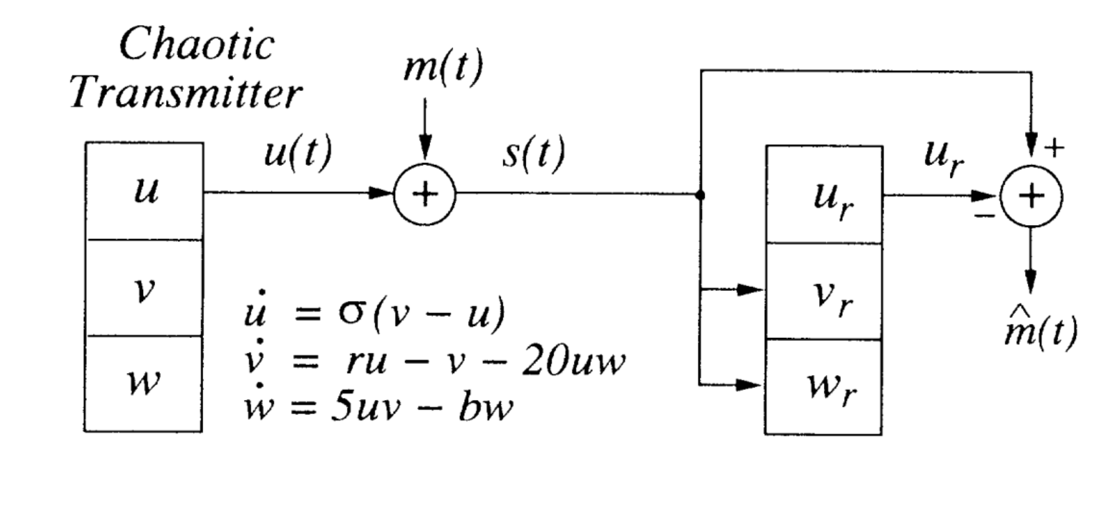

FIG. 5. Chaotic signal masking system.

- Another potential approach to communication applications is based on signal masking and recovery. In signal masking, a noise like masking signal is added at the transmitter to the information-bearing signal m(t) and at the receiver the masking is removed .
- In our system, the basic idea is to use the received signal to regenerate the masking signal at the receiver and subtract it form the received signal to recover m(t).
- This can be done with the synchronizing receiver circuit since the ability to synchronize is robust, i.e., is not highly sensitive to perturbations in the drive signal.

- It is assumed that for masking , power level of m(t) is significantly lower than u(t).
- The Dynamical System implemented at the receiver is ⟶

$$\dot{u}_r = 16(v_r - u_r)\,,$$

$$\dot{v}_r = 45.6s(t) - v_r - 20s(t)w_r\,,$$

$$\dot{w}_r = 5s(t)v_r - 4w_r\,.$$

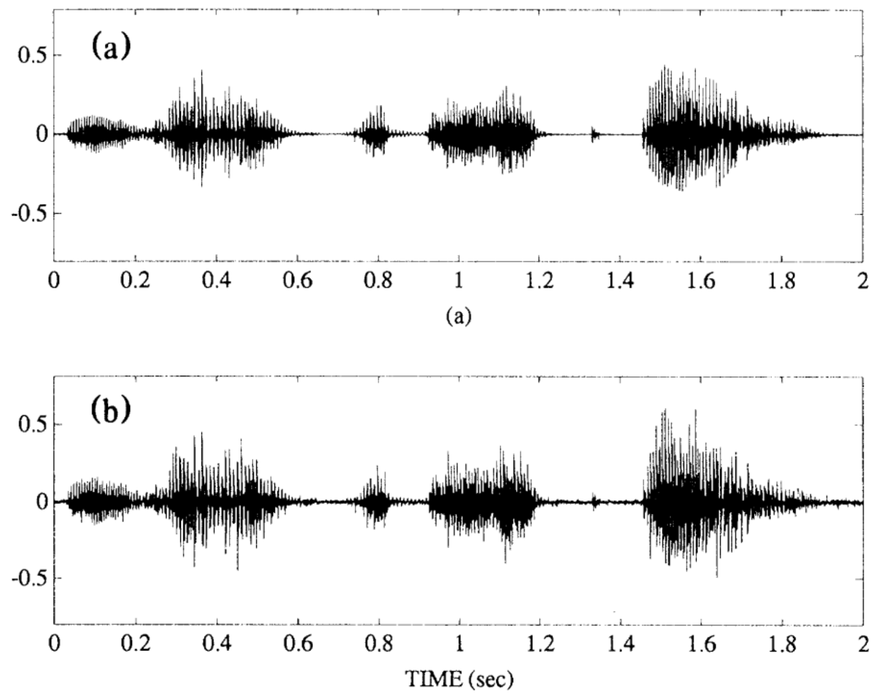

FIG. 6. Circuit data: speech wave forms. (a) Original; (b) recovered.

- If the receiver has synchronized with s(t) as the drive, then u_r(t) $\cong u(t)$ and consequently m(t) is recovered as $m\widehat{(t)} =$ s(t) - u_r(t). Figure 5 illustrates the approach.
- Using the transmitter and receiver circuits, we demonstrate the performance of this system in fig. 6 with a segment of speech from the sentence “He has the bluest eyes”. Figures 6(a) and 6(b) show the original speech, m(t), and the recovered speech signal, $m\widehat{(t)}$, respectively. Clearly the speech signal has been recovered.

REFERENCES:

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Thank you