Dynamics in demand and supply models

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What is supply and demand model ?

- Supply and demand models are among the simplest classes of economic models.
- Simply stated, as the name implies, a supply and demand model is an economic model of the market demand and market supply for a given good or service.
- In particular, a supply and demand model is an effective means of modeling how market forces determine the price, the quantity supplied by producers, and the quantity demanded by consumers, of a good or service.

- In this project we will examine three simple economic models from a mathematical perspective, and examine the dynamics of each.
- Three economic models :

1. Linear supply and linear demand
2. Non linear supply and linear demand
3. Non linear supply and non linear demand

General dynamics of these models :

- In model where there is linear supply and linear demand , the model comes out to be non- chaotic.
- And in non linear supply and linear demand model, as steepness of the S- shape curve is varied, the behavior of the model shifts from the non chaotic to chaotic.
- And in non linear supply and non linear demand model, the behavior of the model remains chaotic throughout.

Linear supply and linear demand model

- For simplicity we consider an economic system with a single good. The model uses the supply and demand curves to determine the price $\mathbf{P_t}$ at time t from the price $\boldsymbol{P_{t-1}}$ at time t - 1. Given an initial price, $\boldsymbol{P_{t-1}}$, the market responds at time t with a quantity Q_t, determined by the supply curve Q_t, = $S(P_{t-1})$. Then the market demand determines the current price $\boldsymbol{P_t}$ by the relation Q_t = $\boldsymbol{D(P_t)}$.

If the supply and demand curves are linear,

$Q_t = S(P_{t-1}) = a + bP_{t-1}$ **(1)**

$Q_t = D(P_t) = c - dP_t$ *(2)*

Then the above two equations reduce to a linear difference equation

$P_t = (c-a)/d - (b/d)P_{t-1}$. *(3)*

For $|b/d| < 1$, the price converges to the equilibrium price.

$P^* = (c - a)/(b + d)$

For $|b/d| = 1$, the price oscillates in a period-2 cycle between P_{t-1} and P_t And for $|b/d| > 1$ the system is unstable and no equilibrium price is achieved.

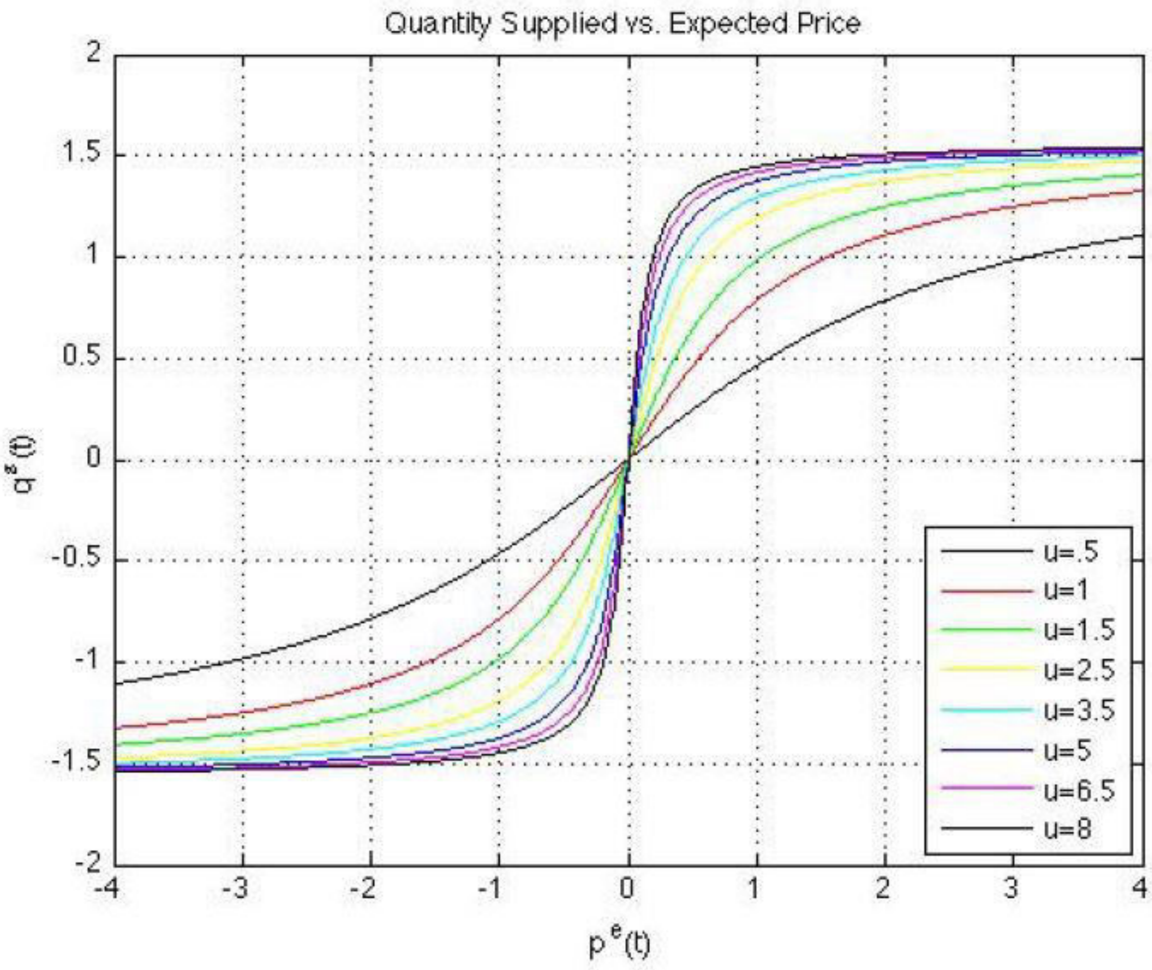
Linear Demand and Non Linear Supply model

- Assumptions-

1. The quantity supplied is assumed to be the function of expected price, and not the actual price.
2. We assumed standard supply curve. The curve exhibits S-Shape.
3. Since arctan function exhibits such a shape, quantity supplied at time t, $q^s(t)$ is expressed as a function of time t and expected price $p^e(t)$ at time t.

$$q^s(t) = \arctan(p^e(t))$$

- Supply is symmetric about the origin and has an inflection point. The steepness of the S shaped supply curve is determined by the parameter μ.



- After defining the supply curve, we define a linear demand curve as

  $q^d(t) = a - bp(t), b>0$

  Where a and b are parameters and p(t) is the actual price at time t.

- Now, we assume that the expected price, $p^e(t + 1)$, at a time $t + 1$ is given as a function of the actual price at t, $p(t)$, and the expected price at t, $p^e(t)$.

That is, the price expectation satisfies

$$p^e(t + 1) = \lambda p(t) + (1 - \lambda)p^e(t)$$

Where λ is a parameter.

- At market equilibrium, we have that the quantity supplied equals the quantity demanded,

  i.e. $q^s(t) = q^d(t)$.

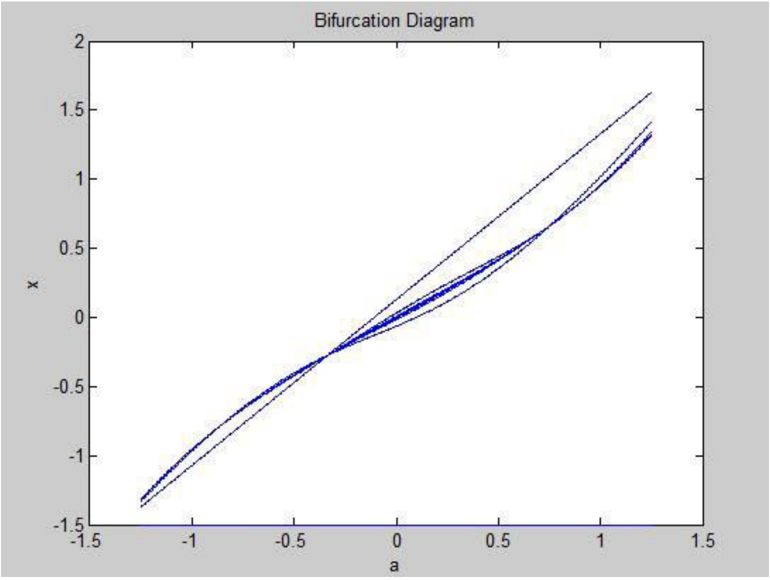
  Thus, equating for these values from above equations, we obtain the following expression for price, in terms of expected price

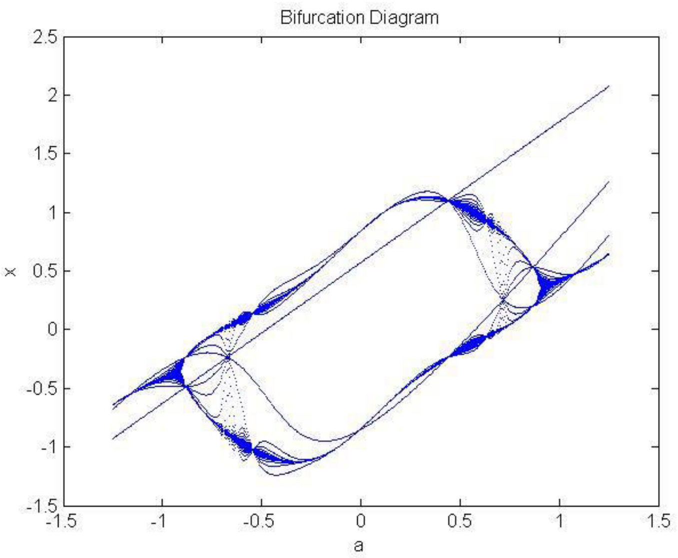
  $p(t) = (a/b) - (\arctan(p^e(t))/b)$

- Substituting the above equality into price expectation equation, we can then express price expectation by the difference equation

  $$p^e(t + 1) = (1-\lambda)p^e(t) + (a\lambda/b) - (\lambda \arctan(p^e(t))/b) \equiv f(p^e(t))$$

Numerical Analysis

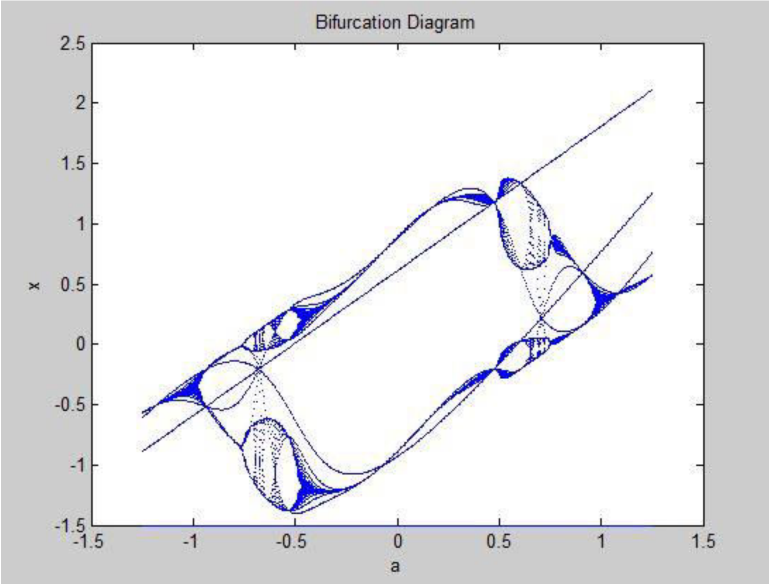


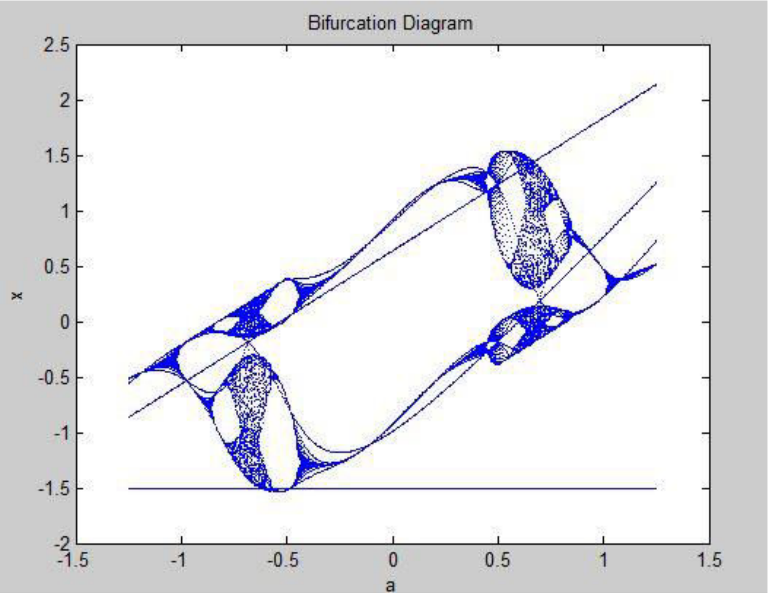


μ=1

μ=3

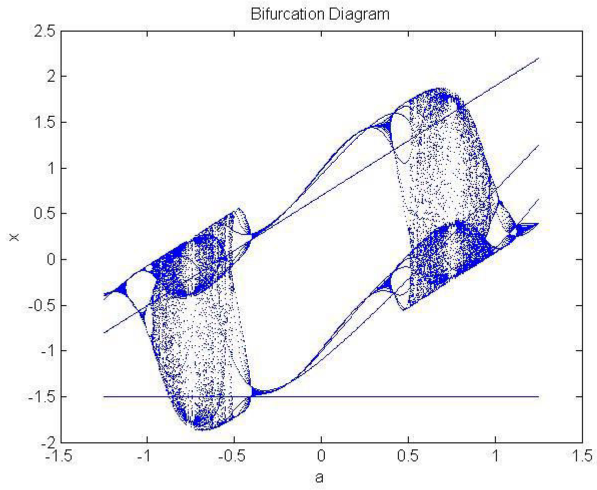
Numerical Analysis

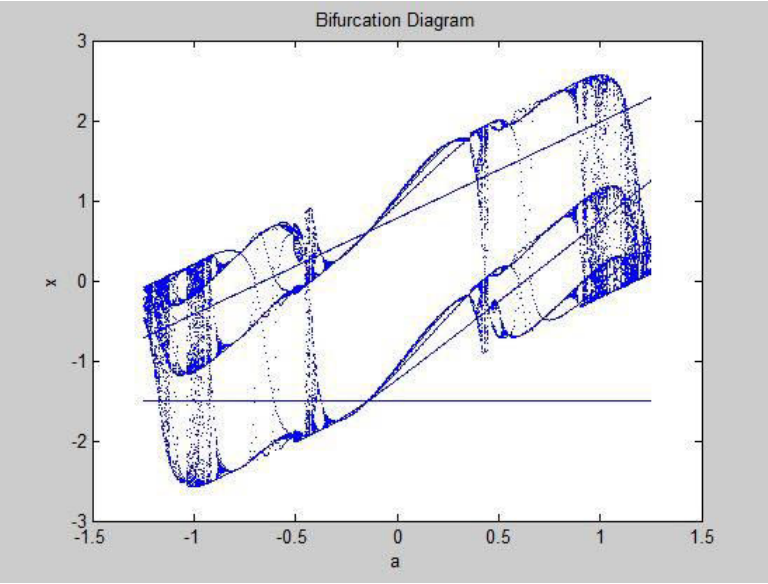




$\mu=3.5$ $\mu=4$

Numerical Analysis



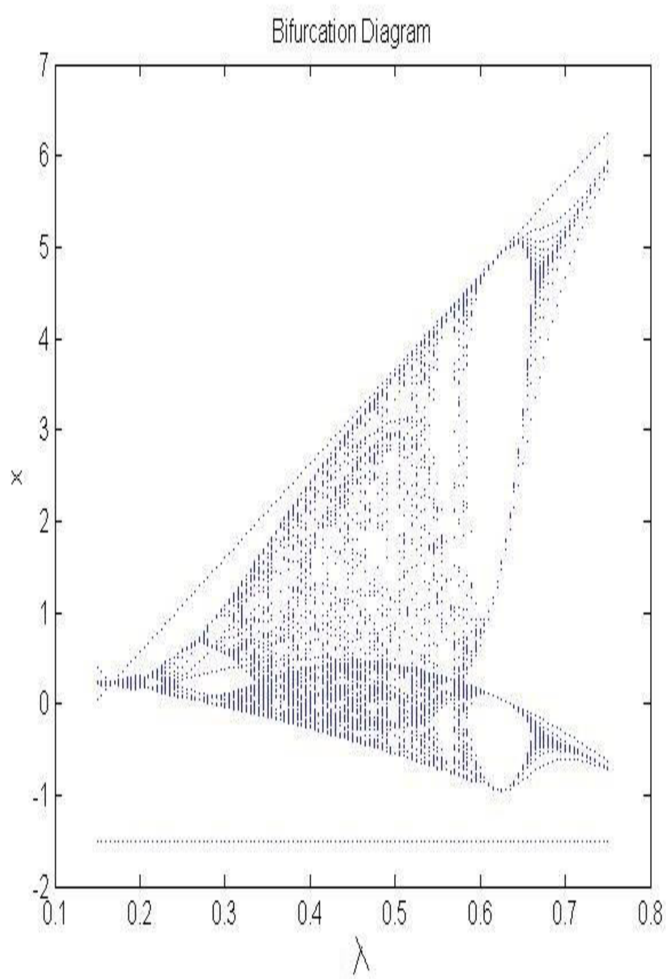


μ=5

μ=15

Numerical Analysis

- As λ is increased, after infinitely many period doubling bifurcations chaotic price behavior arises. As λ is further increased, infinitely many period halving bifurcations appears and chaos disappears and a stable period 2 cycle occur for λ close to 0.75. Roughly, for λ close to 0 and 1, price behavior is regular, while for intermediate values, it is irregular.



Non Linear Demand and Supply

- For the analysis of nonlinear models of supply and demand, we have considered the example of land and housings.
- The price p is usually characterized by the nonlinear inverse demand function of

$$p=a-b\sqrt{Q}$$

  Where a and b are positive constants.
- a is the maximum price in the market, Q is the total quantity in the market.

- Let us consider that price of land and housing at time t is $p_1(t)$ and $p_2(t)$ respectively.

Let $D_1(t)$ is the land demand at time t, $D_2(t)$ is the housing demand at time period t.

Then demands are defined as

$D_1(t)= b_0-b_1p_1(t)+b_2p_1^2(t)$,

$D_2(t)=c_0-c_1p_2(t)+c_2p_2^2(t)$

b_0, b_1, b_2, c_0, c_1, c_2 are constants.

Suppose the Land Supply at time period t, $S_1(t)$ and housing supply at time period t, $S_2(t)$ is defined as

$S_1(t)=e_0+e_1p_1(t)+e_2p_1^2(t)$,

$S_2(t)=d_0+d_1p_2(t)+d_2p_2^2(t)-d_3p_1(t)$

d_0, d_1, d_2, d_3, e_0, e_1, e_2 are positive constants.

According to the law of demand, the slope of demand curve is negative. The slope of supply curve is positive in accordance with the law of supply.

Let Z(p) be the excess demand function descending with price, which denotes the gap between demand and supply, defined as

$Z(p)=D(p)-S(p)$

When the price is low, excess demand exists and when the price is high, excess supply exists.

Substituting the values from above, we get

$Z(p_1(t))=b_0-e_0-(e_1+b_1)p_1(t)+(b_2-e_2)p_1^2(t)$

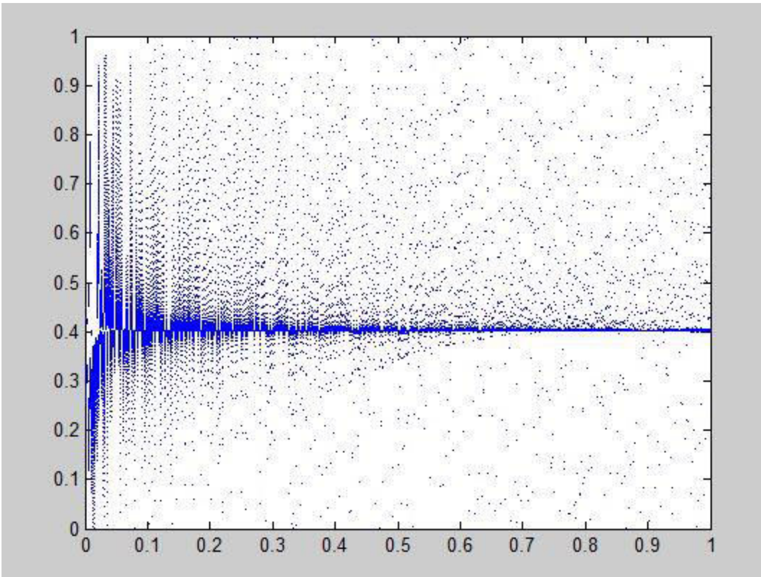
$Z(p_2(t))=c_0-d_0-(d_1+c_1)p_2(t)+(c_2-d_2)p_2^2(t)+d_3p_1(t)$

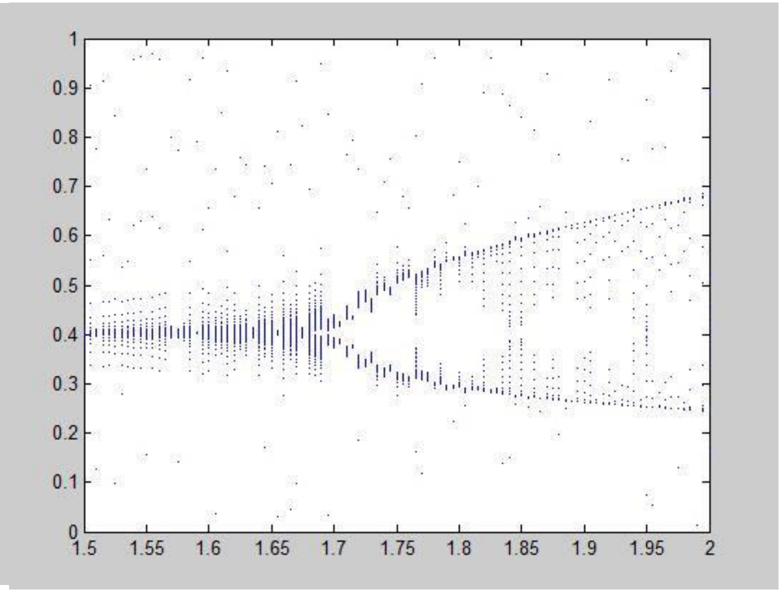
Let α_1 is the adjustment parameter of land prices, which denotes the adjustment degree of benchmark land price controlled by govt. Through the supply plan and α_2 be the adjustment parameter of housing price, then the dynamic model of land price and housing price can be established as follows:

$P_1(t)=p_1(t-1)+\alpha_1 Z(p_1(t-1))$

$P_2(t)=p_2(t-1)+\alpha_2 Z(p_2(t-1))$

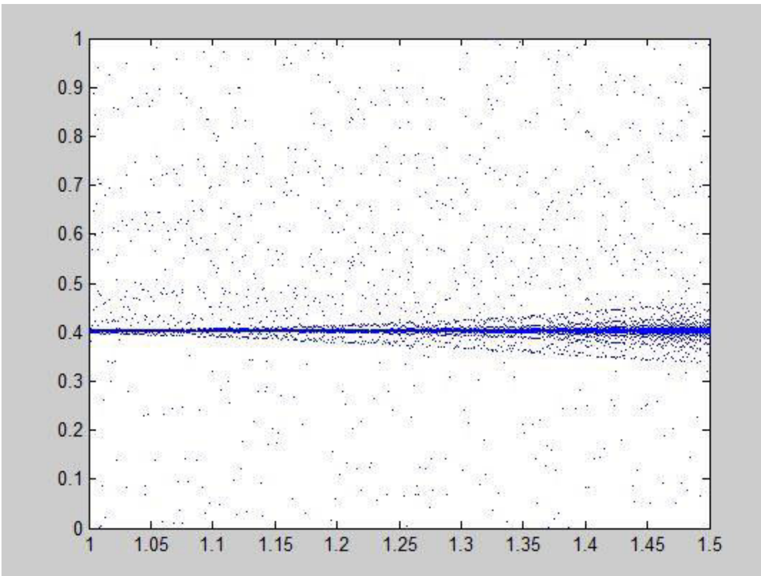
Stability analysis is carried out on these equations.





0<α<1

1<α<1.5


1
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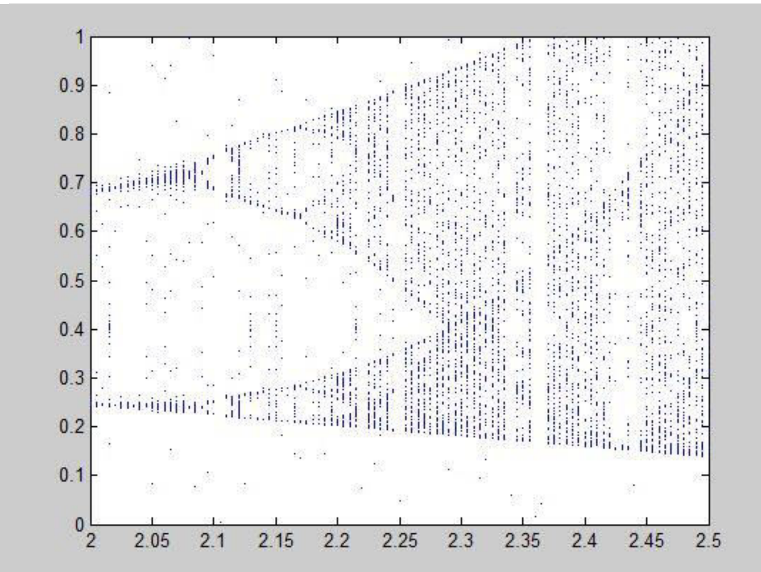
1.5


1
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2
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2.1
2.15
2.2
2.25
2.3
2.35
2.4
2.45
2.5

1.5<α<2

2<α<2.5

Applications

- Analysis is essential and useful for strategic decision makers as it allows both the visualization and control of the states/dynamics of the entire supply and demand chain.
- Bifurcations show various states of supply and demand chain in well specified windows of parameters.
- helps in knowing the price dynamics in the market.

References

- Reasearch article on "Complex dynamics in a nonlinear cobweb model for real estate market" by Junhai Ma and Lingling Mu. (2007)
- Research article on "Dynamics of the cobweb model with adaptive expectation and nonlinear supply and demand" by Cars H. Hommes. (1993)
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11/20/2013

Thank you