Controlling Systems at Chaos

Project by

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What is control of systems at chaos ??

- Chaos occurs at particular value(s) of system parameters

  For example
  The Logistics Equation x(n+1) = mu*x(n)*{1-x(n)}
  At mu=4 the system behaves as chaotically

- When the systems behave chaotically we call it system at chaos

- We can control the systems at chaos using some methods and techniques.

  This is called the control of systems at chaos.

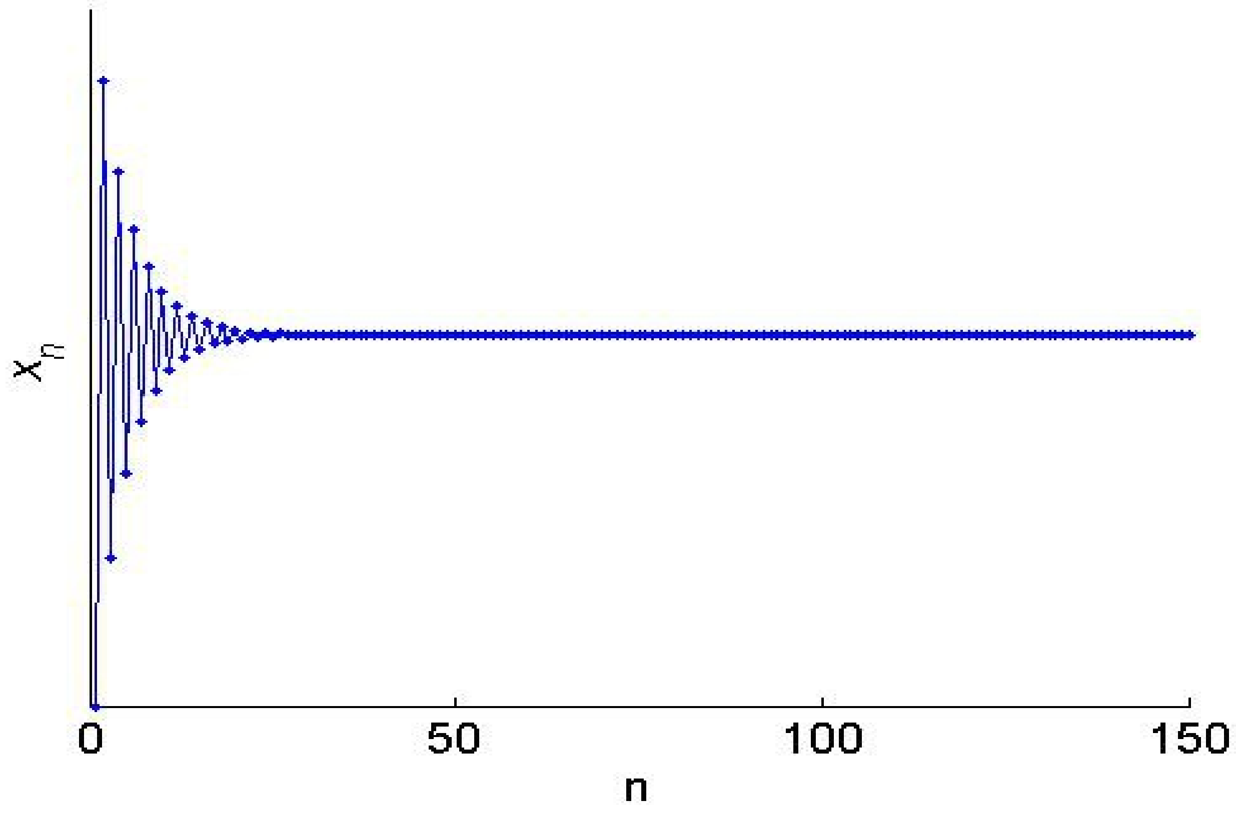
Why do we need to control the systems at chaos ??

- In the past the scientists have attempted to remove chaos from systems.
- But in the last decade the potential uses of systems showing chaos have been realized.
- For systems which can show chaos are considered better.
- Chaos can be controlled in a better way.

Stability Good, Chaos Bad

Stability Good, Chaos Better

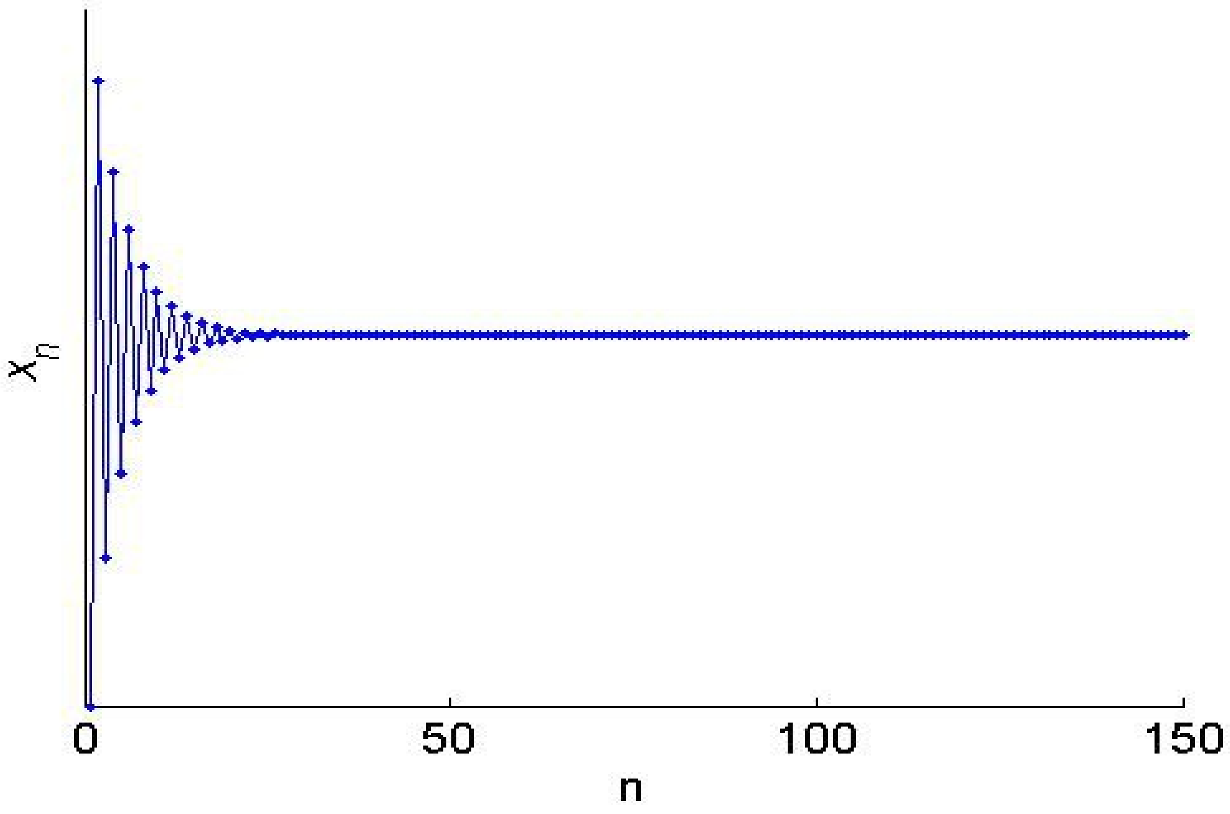
- Let's see how



We took mu=2.8 and x(1)=0.6

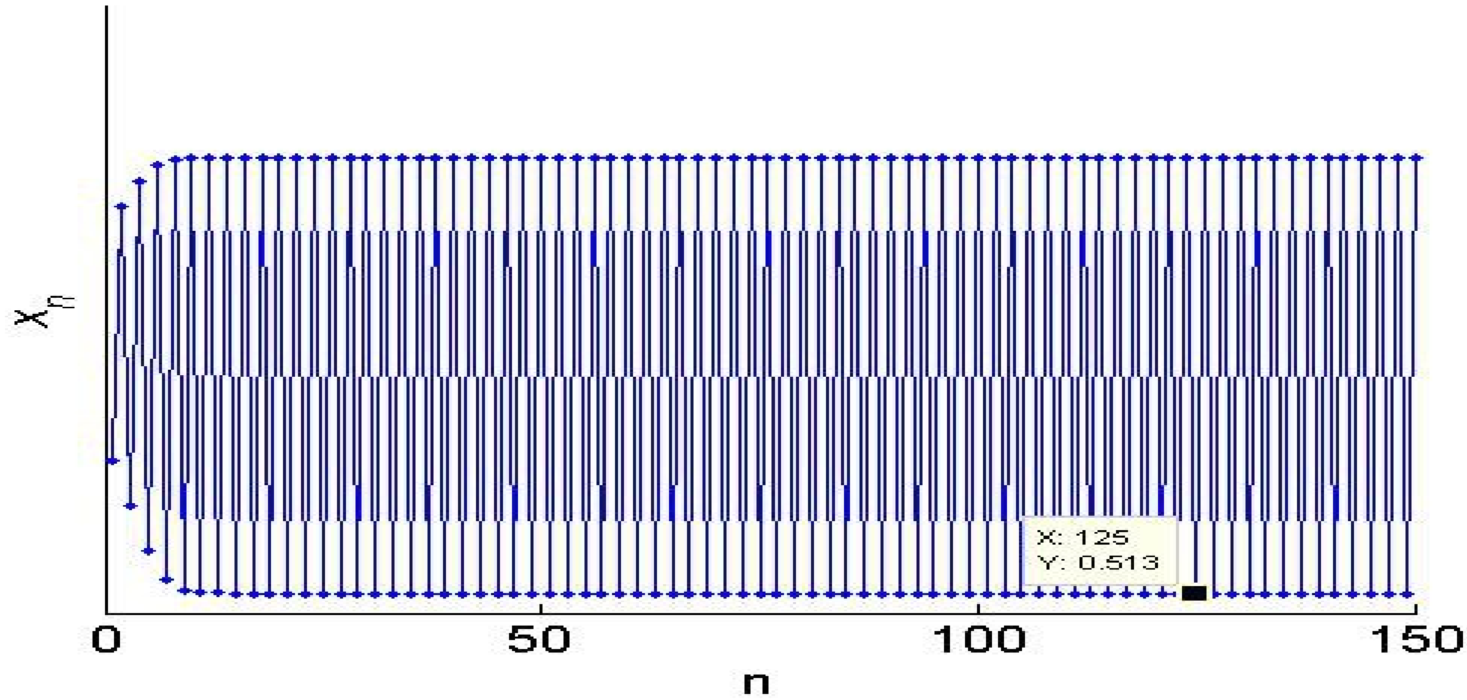
Here the x(s)=0.6429

Can we change x(s) in this system??



We took mu=2.8 and x(1)=0.6
Here the x'=0.6429
Can we change x' in this system??

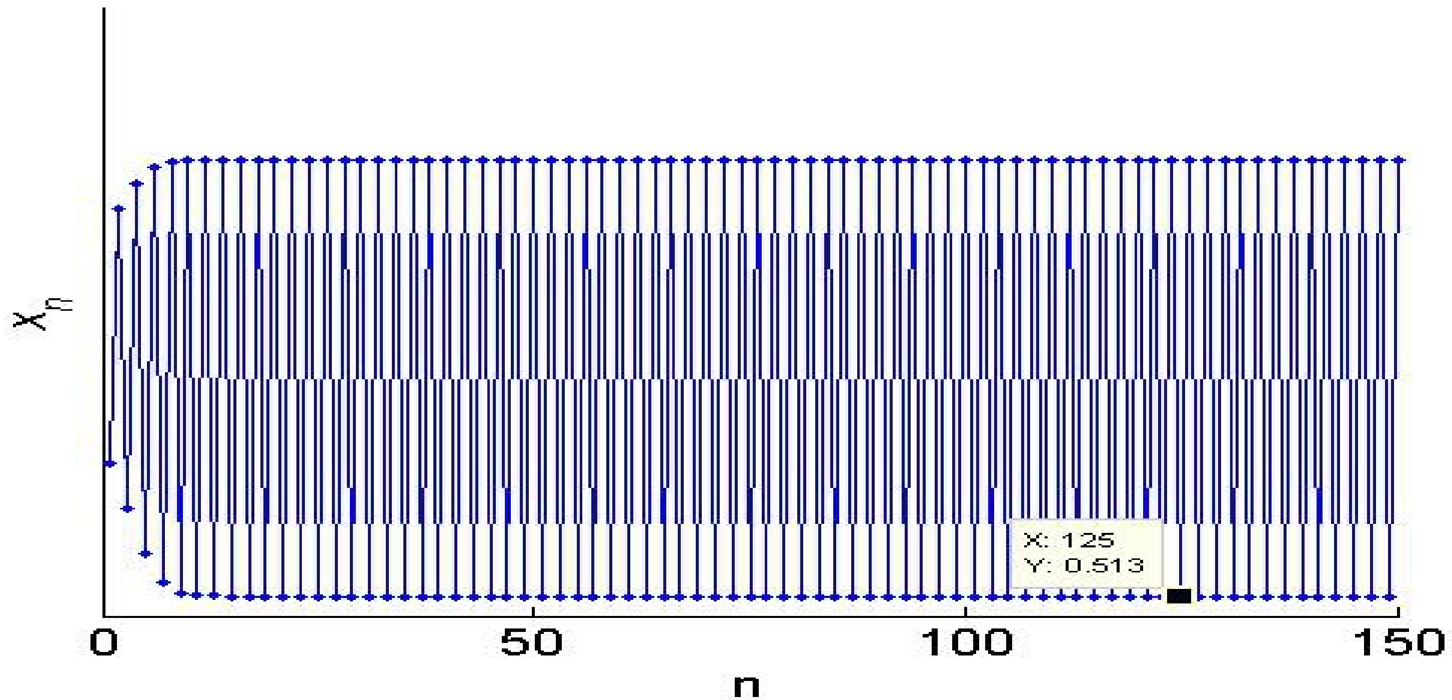
no



We took mu=3.2 and x(1)=0.6

Here the x'=0.513 and x“=0.7995

Can we change fixed points in this system??



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Can we change fixed points in this system??

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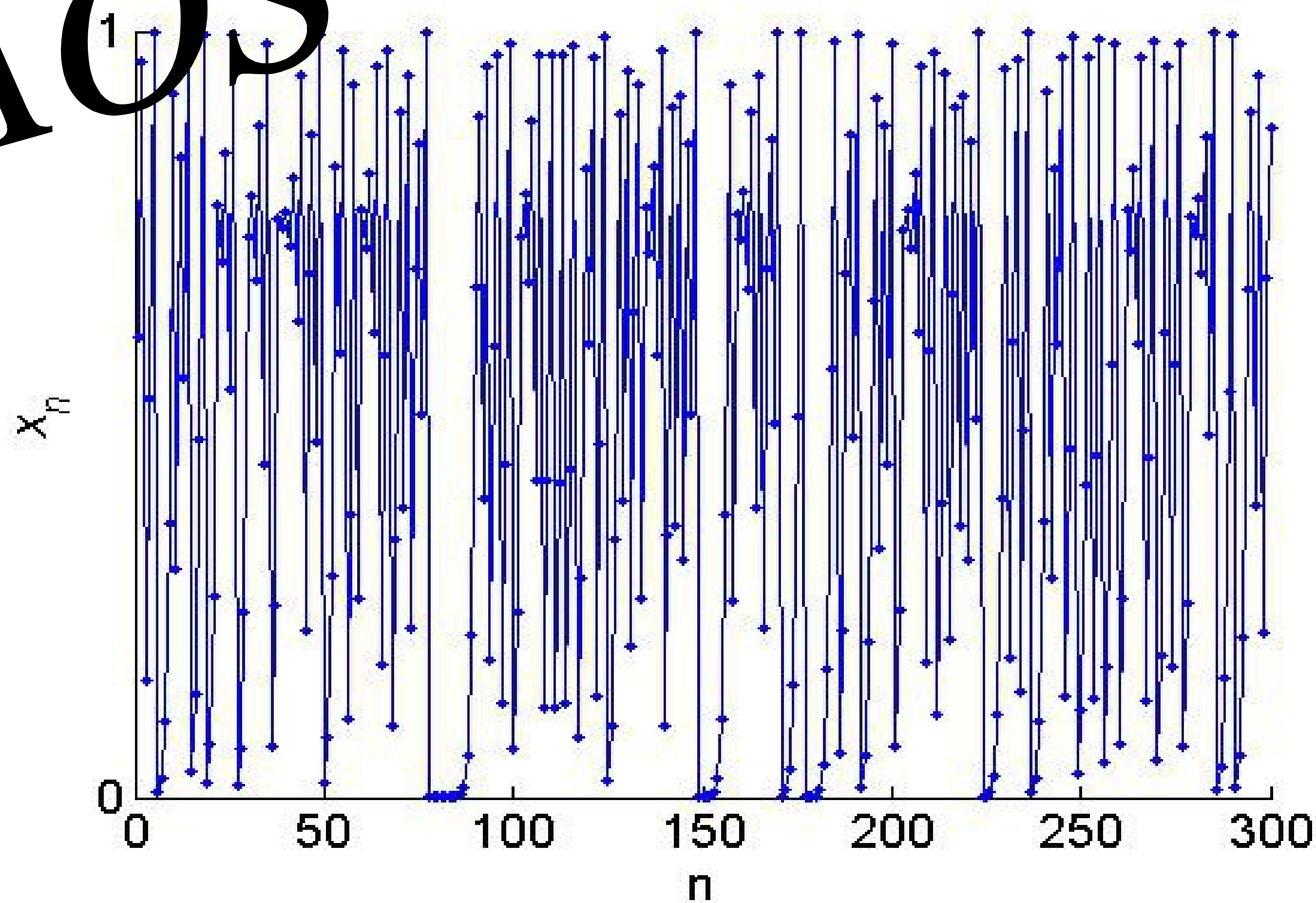
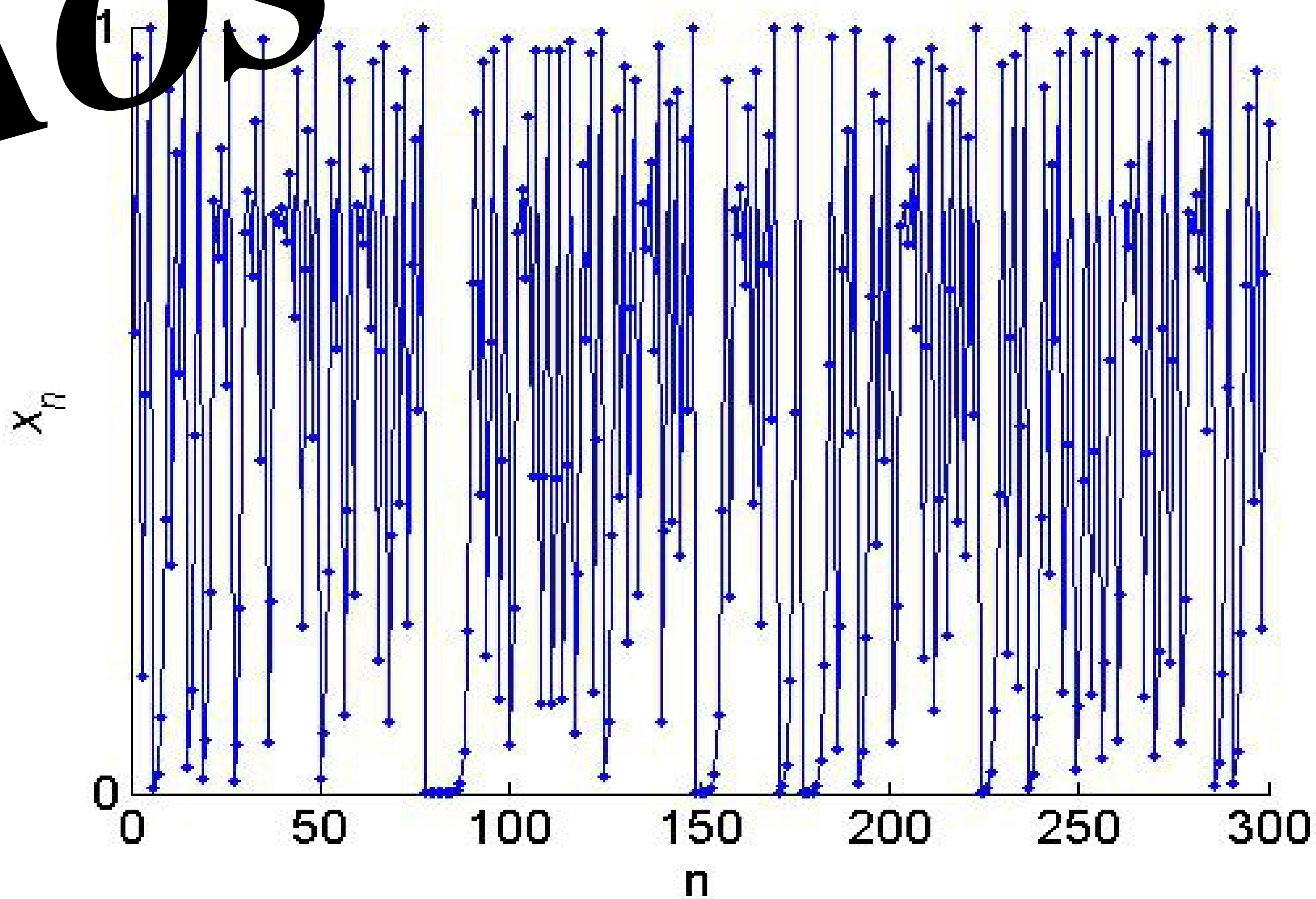
CHAOS



Here mu=4 and x(1)=0.6

CAN WE CONTROL THIS SYSTEM AT CHAOS ???

CHAOS



Here mu=4 and x(1)=0.6

CAN WE CONTROL THIS SYSTEM AT CHAOS ???

yes

We can't change mu. Because mu is a system parameter.

So in non-chaotic systems we can't change the fixed points.

But

In chaotic systems we can have one or more fixed points according to our wish and can change them.
For this we need the use of proportional periodic pulses.

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BUT HOW ??

Consider logistic map

$$x(n+1)=f(x(n))=mu*x(n)\{1-x(n)\}$$

Now for controlling the chaos we need to apply instantaneous pulses (k) to the system
once in every p iterations

$$x(i) \longrightarrow k*x(i)$$

For fixed points we need

$$x(s)=f(x(s))$$

And the fixed point will be stable if $\left|\frac{d\, f(x^*)}{dx}\right| < 1$

In the new system the fixed point will be at $k\{f(x^*)^p\}=x^*$

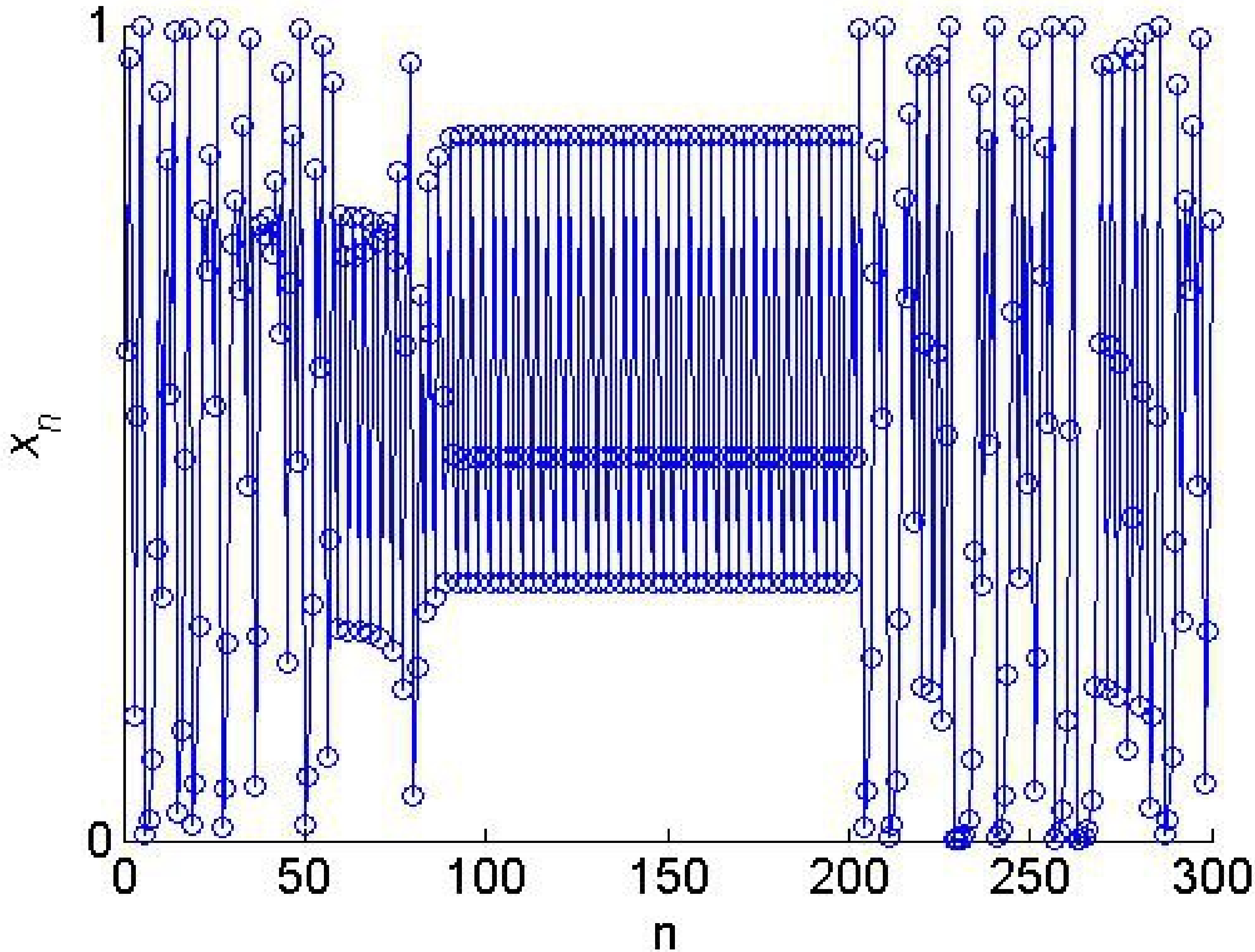
So the stability condition will be $\left|k\,\frac{d\{f(x^*)^p\}}{dx}\right| < 1$

So lets define $c(p,x)=\frac{x^*}{f(x^*)^p}\;\frac{d\{f(x^*)^p\}}{dx}$

So now the condition for stable fixed point becomes $\left|c(p,x)\right| < 1$

Let's see how it works in MATLAB

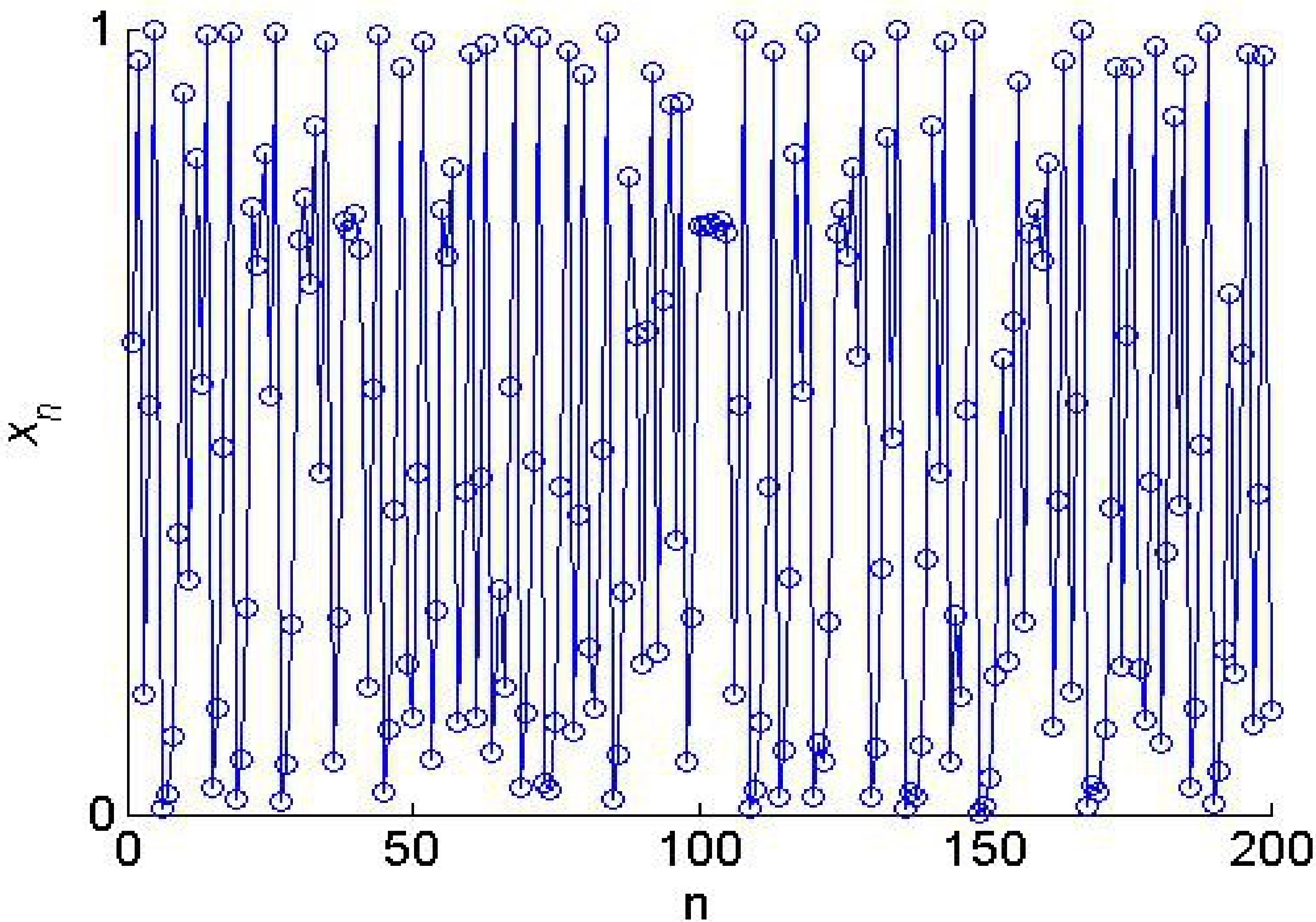
```
clear
itermax = 300;
mu = 4;
k=.317;
x(1)=.6;
for n=1:3:itermax
    x(n+1)=mu*x(n)*(1-x(n));
    x(n+2)=mu*x(n+1)*(1-x(n+1));
    x(n+3)=mu*x(n+2)*(1-x(n+2));
    if n>60 & n<200
        x(n+1)=k*mu*x(n)*(1-x(n));
        x(n+2)=mu*x(n+1)*(1-x(n+1));
        x(n+3)=mu*x(n+2)*(1-x(n+2));
    end
end
hold on
plot(1:itermax,x(1:itermax))
plot(1:itermax,x(1:itermax),'o')
fsize=15;
set(gca,'xtick',[0:50:itermax],'FontSize',fsize)
set(gca,'ytick',[0,1],'FontSize',fsize)
xlabel('n','FontSize',fsize)
ylabel('\itx_n','FontSize',fsize)
hold off
```



1
0
x_n
0
50
100
150
200
250
300
n

Now Lets try for period 8

```
clear
itermax = 200;
mu = 4;
k=.2;
x(1)=.6;
for n=1:8:itermax
    x(n+1)=mu*x(n)*(1-x(n));
    x(n+2)=mu*x(n+1)*(1-x(n+1));
    x(n+3)=mu*x(n+2)*(1-x(n+2));
    x(n+4)=mu*x(n+3)*(1-x(n+3));
    x(n+5)=mu*x(n+4)*(1-x(n+4));
    x(n+6)=mu*x(n+5)*(1-x(n+5));
    x(n+7)=mu*x(n+6)*(1-x(n+6));
    x(n+8)=mu*x(n+7)*(1-x(n+7));
    if n>40
        x(n+1)=k*mu*x(n)*(1-x(n));
        x(n+2)=mu*x(n+1)*(1-x(n+1));
        x(n+3)=mu*x(n+2)*(1-x(n+2));
        x(n+4)=mu*x(n+3)*(1-x(n+3));
        x(n+5)=mu*x(n+4)*(1-x(n+4));
        x(n+6)=mu*x(n+5)*(1-x(n+5));
        x(n+7)=mu*x(n+6)*(1-x(n+6));
        x(n+8)=mu*x(n+7)*(1-x(n+7));
    end
end
hold on
plot(1:itermax,x(1:itermax))
plot(1:itermax,x(1:itermax),'o')
fsize=15;
set(gca,'xtick',[0:50:itermax],'FontSize',fsize)
set(gca,'ytick',[0,1],'FontSize',fsize)
xlabel('n','FontSize',fsize)
ylabel('\itx_n','FontSize',fsize)
hold off
```



Not working for period eight.............????

Observations

- The higher the period we want the more no. of iterations it takes to stabilize.

- This proportional pulses technique in systems is applicable to low periods. In the previous slide we saw that we could not stabilize period eight pulsed logistic map.

Let's find out the reasons for those observations that we got

Consider the control parameter

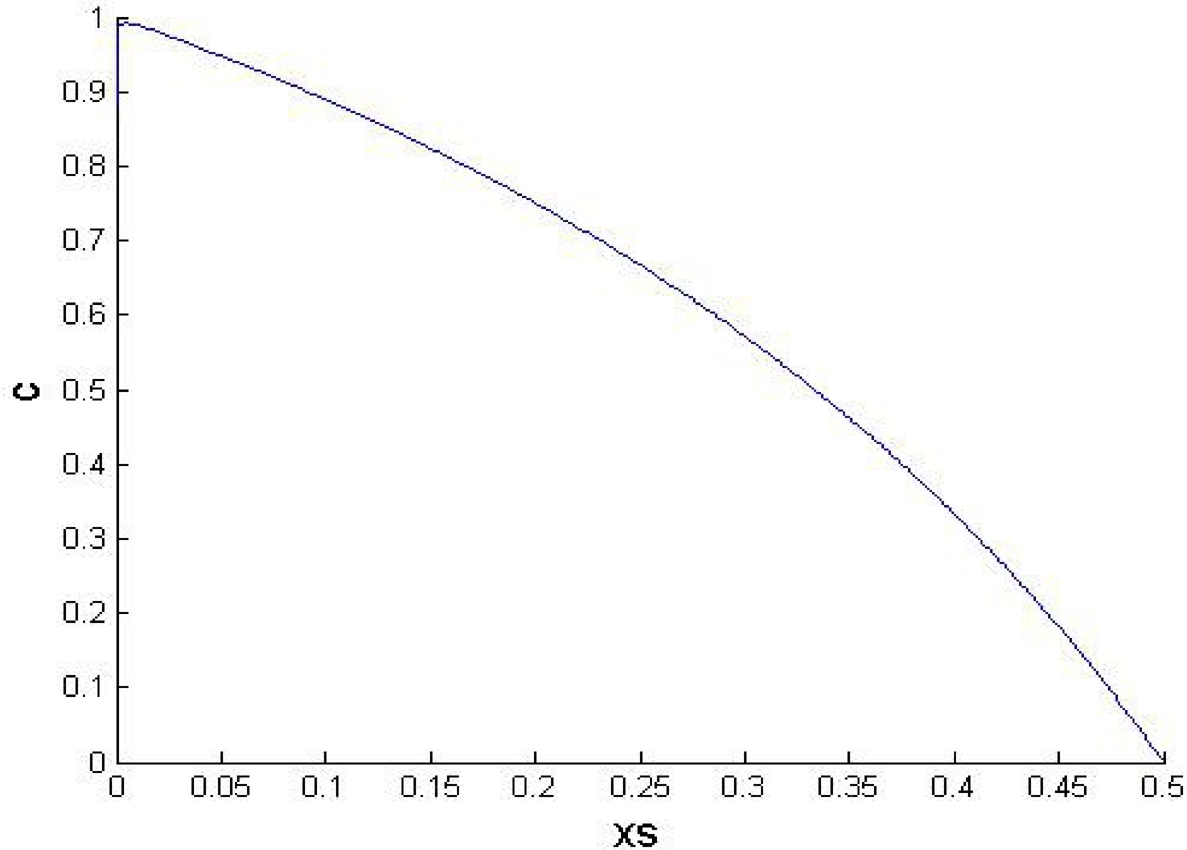
$$c(p,x) = \frac{x^*}{f(x^*)^p} \frac{d\{f(x^*)^p\}}{dx}$$

Can we plot control parameter vs fixed points ??

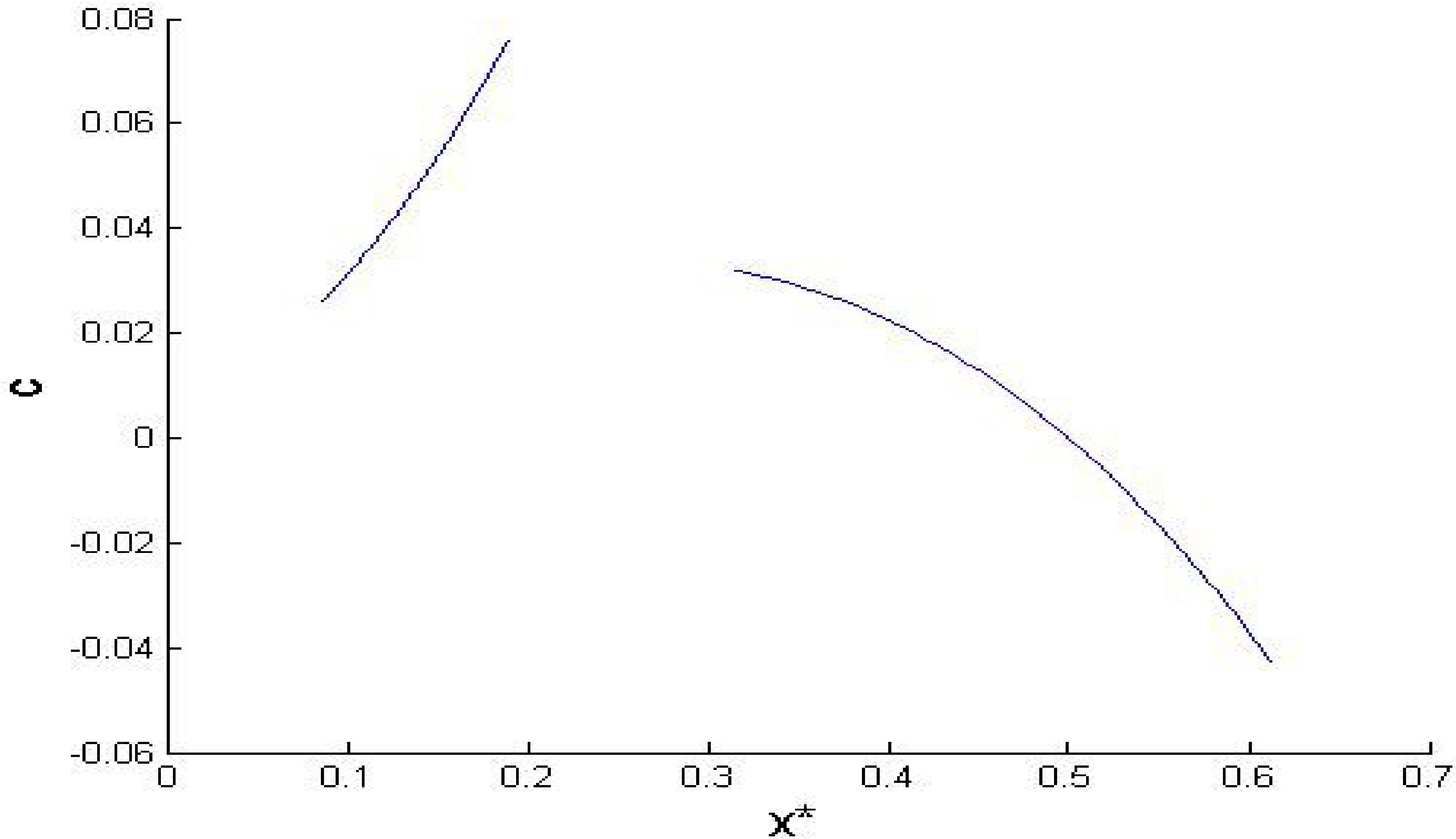
Let's plot control parameter vs fixed points graphs

For 1-period one pulsed map
2-period two pulsed map
3-period four pulsed map
....
....

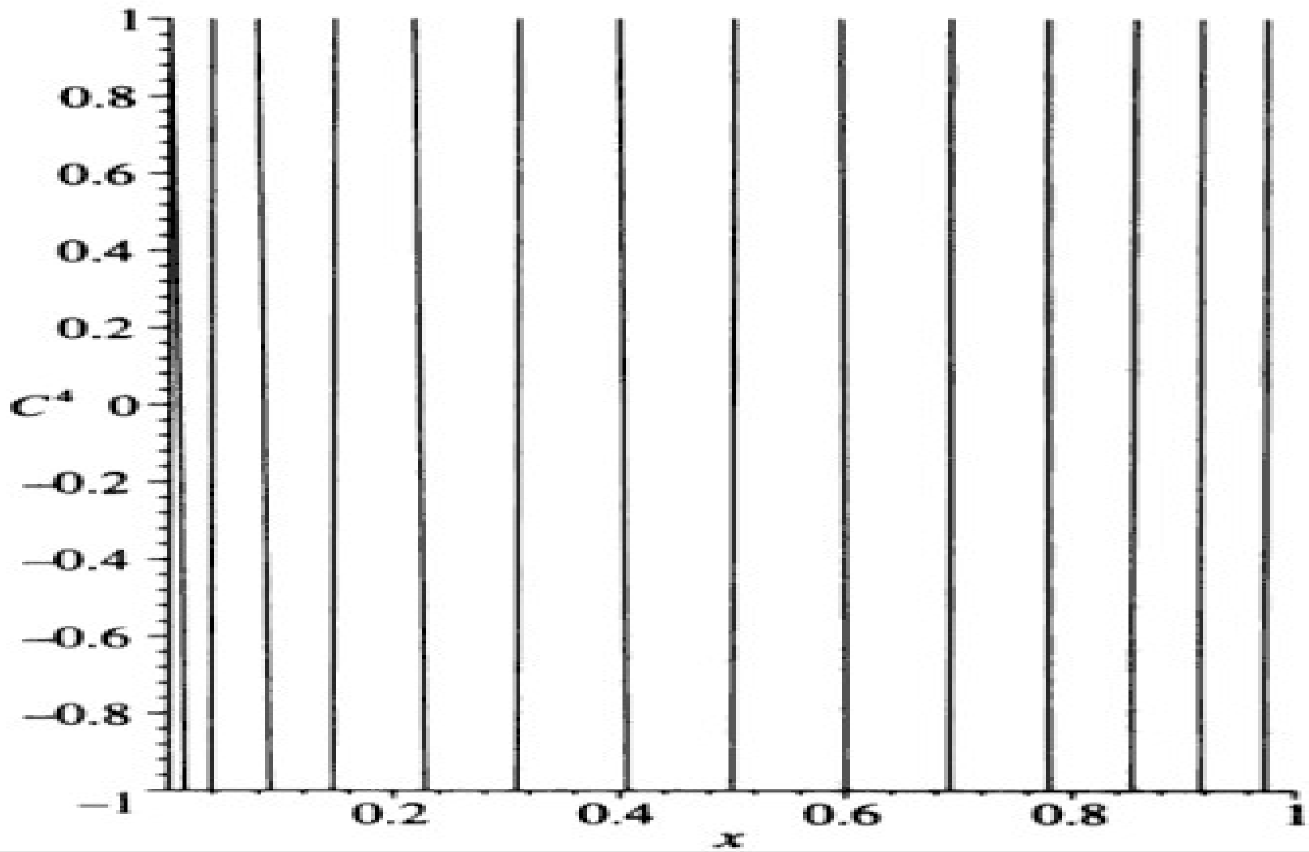
```
clear
itermax = 300;
mu = 4;
k(1)=.05;
x(1)=.6;

for m=1:1:450
for n=1:1:itermax
    x(n+1)=mu*x(n)*(1-x(n));
    if n>60
        x(n+1)=k(m)*mu*x(n)*(1-x(n));
    end
end
c(m)=k(m)*mu*(1-2*x(itermax-1));
xs(m)=x(itermax-1);
k(m+1)=k(m)+ 0.001;
end
hold on
plot(xs(1:450),c(1:450))
fsize=15;

xlabel('xs','FontSize',fsize)
ylabel('c','FontSize',fsize)
hold off
```



Control parameter vs fixed point(x*) with period one pulsing



Control Parameter vs fixed points(x*) with period two pulsing



Control Parameter vs fixed points(x*) in period 4 pulsed map

We found out from the control parameter graphs that when increase the period of the pulses we get more and more no of nearly vertical lines.

In period eight pulsing there many more lines in the graph. Which directly implies there are many points with same control parameter which indirectly implies that evening after pulsing the period 8 pulsed map will show chaos.

So now we got to know that we can stabilize chaos using low period pulsing technique only

But how to apply this pulsing in real world ?

A small example will help us understand better.

Fisherman and pond with fishes

Once in every 2 weeks the fisherman comes to the pond and catches exactly 50 percent of the total fishes in the pond.
This is equivalent to giving an external proportional pulse of 0.5

Conclusion

Stability Good
But
Chaos is better

References

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- T. Kapitaniak, *Controlling chaos: Theoritical and practical methods in Non-Linear Dynamics*, Academic Press,New York,1996