

# Effect of the Slot Position on the Response of Slot Microresonators: Numerical Investigation

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## ABSTRACT

Due to unique feature of high electric field confinement in low refractive index slots, microresonators based on cavities with a slot were investigated for novel sensing and nonlinear applications. The resonance properties of such slot resonators are significantly affected by the position of the slot inside the cavity. In this work we investigate this dependence numerically. Consistent handling of discontinuous fields and efficient resolution of sub-wavelength slot pose a numerical challenge in simulation of slot microresonators. We address these difficulties by modelling the slot resonators using coupled mode theory. Using this model, effects of the slot position on the resonances of the slot microresonators are analyzed.

**Keywords:** Integrated optics, slot waveguides, slot microresonators, coupled mode theory.

## 1. INTRODUCTION

Recently it has been shown that by introducing a sub-wavelength slot in conventional waveguides, one gets high electric field confinement in low refractive index slot [1]. This property originates from the discontinuity of the normal component of electric field across the material interface. By introducing such a slot in the cavity of microresonators, one can further boost the resonance field enhancement. These slot resonators have been demonstrated for novel sensing applications [2].

Analysis of the bent slot waveguides showed that the position of the slot inside the guiding core greatly influences the modal field properties of the bent/curved waveguides [3]. This refers to bending loss, field peak localization, power confinement in various regions of the waveguide, etc. Thus when the slot is introduced in the cavity of a microring resonator, it is expected that the position of the slot affects the resonance wavelengths and the Q-s of the resonator.

Numerical investigation of such effect is a challenging task. While simulating slot resonators, underlying numerical method must consistently handle the discontinuous fields, efficiently resolve the sub-wavelength slot, and keep under control any discretization errors in representation of curved surfaces. For direct numerical simulations with a method like finite difference time domain method, it results into high computational cost due to fine spatial mesh and long computational time. It has been shown that the spatial coupled mode theory (CMT) based modelling of resonators overcomes some of these obstacles, and offers further insight in the functioning of the resonators [4]. In this contribution, we analyze the slot resonators by using this CMT approach, and investigate the effect of slot position on the resonances of the slot resonators.

## 2. COUPLED MODE MODEL OF THE SLOT RESONATORS

Consider a resonator made up of a ring cavity with a sub-wavelength slot as shown in Fig.1.

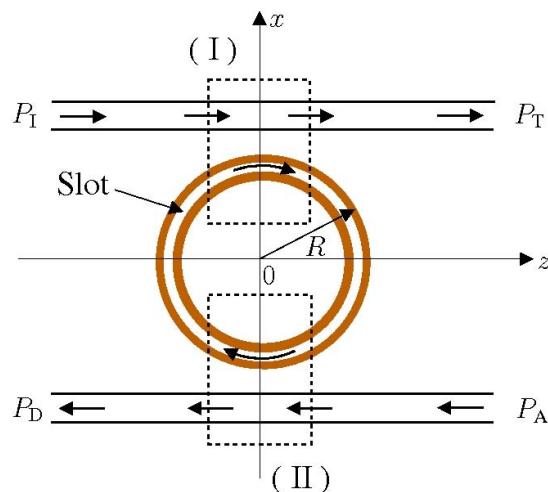


Figure 1. Schematics of the functional decomposition of the slot microresonators.

We consider frequency domain, linear optics, and 2-D setting in the XZ plane with no material/field variation in the y-direction. The CMT model is based on functional deposition of the resonators into two bent-straight

waveguide couplers (I) and (II) with appropriate connections by the segments of bent and straight waveguides [5]. The constituent couplers are assumed to be adiabatic and have finite interaction zone. Moreover, we consider a symmetrical setting with respect to the axes  $x$  and  $z$ , i.e. the two couplers are identical.

The response of the couplers is characterized by the coupler scattering matrices, whereas the field propagation along the ring segments is characterized in terms of the bent mode propagation constants. As shown in Ref. [4], the transmitted power  $P_T$  and the drop power  $P_D$  can be computed in terms of the coupler scattering matrices and the propagation constants. Next we outlined the procedure to obtain these required quantities.

The required 2-D bent slot mode propagation constants are calculated using analytic model of the bent-slot waveguides [3]. This model is based on analytic representation of the modal fields in terms of Bessel and Hankel functions for a bent slot waveguide with a piecewise constant, radial refractive index profile. Matching the fields at the material interfaces, along with bounded outgoing solution requirement, one gets the dispersion equation for bent slot waveguides. Solving this equation numerically, one obtains the required bent slot modes and their propagation constants. In the present 2-D setting, transverse magnetic (TM –also known as E) polarization gives the slot field enhancement [3]. For this the  $y$  component of the magnetic field is the principal field component, and the radial component of the electric field is the discontinuous normal component.

Capitalizing on the availability of the analytic bent slot modes, the bent slot-straight waveguide couplers are modelled using the spatial coupled mode theory [4]. In this frequency domain CMT approach, the interaction is restricted to a finite coupling region, and outside this region the fields are assumed to be uncoupled. Within the coupler, the coupler field is approximated by a linear combination of the modal fields of uncoupled bent-slot waveguide and straight waveguide, with *a priori* unknown amplitudes. The governing equations for these amplitudes are obtained by inserting the coupled field ansatz in the variational formulation for Maxwell equations [4]. As a necessary condition for stationarity of the variational formulation, one gets the coupled mode equations. Solving these equations numerically over a prescribed coupler region, we get the coupler transfer matrix, which associates the initial coupled amplitudes to the final coupled amplitudes. As explained in Ref. [4], the coupler field is projected on to the modes of the straight waveguides to obtain the required scattering matrix. This projection operation is essential to get the stable amplitudes for the straight waveguides modes outside the coupler. It also vindicates the aforementioned finite coupler region assumption.

For a given wavelength  $\lambda$ , by the above outlined procedures, we are in a position to evaluate the parametric model of slot microresonators. Repeating the computations for series of wavelengths gives the spectral response of the slot resonator. There also exists speed up techniques for spectrum computations, see Ref. [4].

### 3. SIMULATION RESULTS

We consider the simulation setting as following: A microring of width  $w_{\text{tot}} = 1 \mu\text{m}$  has a slot of width =  $0.2 \mu\text{m}$  and refractive index 1. The slot is introduced between two ring-cavity layers of refractive index 2.1 and widths  $\eta w$  (for inner layer) and  $(1 - \eta)w$  (for outer layer) respectively, with  $w = 0.8 \mu\text{m}$ . Here  $\eta$  is the asymmetry parameter controlling the position of the slot inside the ring core [3]; ring radius (= the outermost cavity interface) =  $5 \mu\text{m}$ , straight waveguides are made up of core of refractive index = 2.1 and width =  $0.4 \mu\text{m}$ , and the background refractive index = 1, gap widths between the cavity and the bus waveguides =  $0.4 \mu\text{m}$ . The couplers are simulated on the domain  $X = [1, 8] \mu\text{m}$ ,  $Z = [-3, 3] \mu\text{m}$ ,  $h_x = 0.005 \mu\text{m}$ ,  $h_z = 0.1 \mu\text{m}$ .

The resultant TM mode spectral response for various values of asymmetry parameter  $\eta$  is shown in Fig. 2. Each dip in the through power corresponds to the resonance of a particular TM cavity mode. The prominent sharp dips correspond to the resonances of the fundamental cavity modes  $\text{TM}_0$ . For  $\eta = 0.5$ , one also sees minor secondary dips, which correspond to higher order cavity mode -- in the present case the first order  $\text{TM}_1$  mode. The spectral response also hints that the resonances corresponding to  $\eta = 0.7$  are very sharp compared to those corresponding to  $\eta = 0.4$  and  $\eta = 0.5$ .

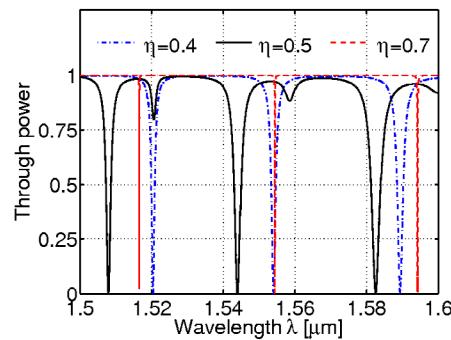


Figure 2. Illustration for the effect of the slot position on the resonator response. The results are obtained with the CMT model of the slot resonators, for the resonator setting in Sec. 3.

To get a closer perspective of the various resonances in Fig. 2, we looked into the corresponding resonance field plots. As a case in point, we traced the resonance corresponding to  $\text{TM}_{0,30}$  (the fundamental TM mode with the angular mode number 30). Fig. 3 shows the effect of the slot position on the  $\text{TM}_{0,30}$  resonances. It shows that the resonance wavelengths are significantly affected by the position of the slot inside the ring core.

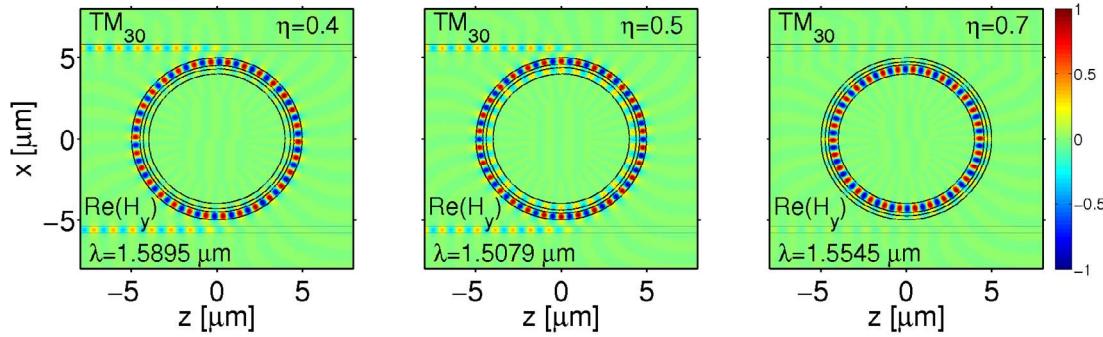


Figure 3. Resonance field plots for the setting as in Sec. 3.

For  $\eta = 0.4$ , the principal field component  $H_y$  is neatly localized in the outer most high index layer of the ring cavity. When the slot is placed symmetrically in the core of the ring, i.e.  $\eta = 0.5$ , even though the significant part of the  $H_y$  is in the outer most layer, the field is a non-negligible in the inner most high index layer. But for the case  $\eta = 0.7$ , the field is localized in the inner most high index layer. Due to such a tight confinement of the field, as mentioned previously, the corresponding resonances for  $\eta = 0.7$  are sharper.

The  $Q$  values for  $\eta = 0.4, 0.5$  and  $0.7$  are approximately 900, 1200, and 26000 respectively. This shows that by positioning the slot at the appropriate place inside the cavity, the  $Q$  value can be optimized.

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