Lab 03 - Population growth model with critical threshold

Consider the population of a biological species with a certain carrying capacity which goes to extinction if the initial population is below a certain number *i.e.* there exists a threshold population for the species to survive. The features of such a population dynamics are:

- **Upper limit on the population based on the carrying capacity**
- **Exponential growth at initial stages and saturation at later stages**
- **Extinction when the initial population is less than the threshold population**

The system is modelled using the following equation:

$$\frac{dx}{dt} = -ax\left(1 - \frac{x}{\lambda_1}\right)\left(1 - \frac{x}{\lambda_2}\right) \tag{1}$$

λ_1: carrying capacity; λ_2: threshold population; $0 < \lambda_2 < \lambda_1$

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- **For $a = 0.12\ \text{min}^{-1}$, $\lambda_1 = 1000$ and $\lambda_2 = 50$, sketch the population growth in 10 h considering the follwing initial populations:**
  - $x(0) = 25$
  - $x(0) = 50$
  - $x(0) = 250$
  - $x(0) = 1000$
  - $x(0) = 1250$

- **Considering $x(0) = 250$, study the effect of "a" on the population dynamics.**

- **Sketch the phase portrait for the system. Analyse whether the system has a bifurcation about $a = 0$.**