Homework 1 Solutions

H1.1 The workfunction of Barium and Tungsten metal are 2.5 eV and 4.2 eV, respectively. Can these metals be used in photovoltaic cells which converts light energy to electrical energy?

Solution:

Photovoltaic cells absorb visible light which is in the range 4000-7000 Å.

Energy of light photon with λ = 4000 Å,

$$E = \frac{hc}{\lambda} = \frac{6.626*10^{-34}*3*10^{8}}{4000*10^{-10}} = 3.106eV$$

Energy of light photon with λ = 7000 Å,

E = 1.77eV.

Work function of Tungsten is 4.2eV which is above than visible range. Hence only Barium can be used in photovoltaic cells.

H1.2 A photon of wavelength 40 nm strikes an electron at rest and is scattered at an angle of 150^{o}. Find the wavelength of the scattered photon.

Solution:

Compton shift, $\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - cos\theta) = \frac{6.626*10^{-34}}{9.1*10^{-31}*3*10^{8}}(1 - cos150^{o})$

$$\Delta\lambda = 0.045Å$$

Wavelength of the scattered photon, $\lambda' = \lambda + \Delta\lambda$

$$\lambda' = 4.045Å$$

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H1.3 What is the maximum wavelength of light that can be absorbed by Hydrogen atom in the ground state? What will be the next smallest wavelength that will be absorbed by the same atom?

Solution:

When H atom is in the ground state, n=1

Maximum wavelength corresponds to minimum energy which should be the n=1 → n=2 transition.

Hydrogen atom energy, $E_n = \frac{-13.6}{n^2}\ eV$

$$E_1 = -13.6\ eV$$

$$E_2 = -3.4\ eV$$

$$\Delta E = E_2 - E_1 = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E_2 - E_1} = \frac{6.626*10^{-34}*3*10^{8}}{10.2*1.602*10^{-19}}$$

$$\lambda = 122nm$$

Next smallest wavelength should correspond to $E_1 \rightarrow E_3$

$$\lambda = \frac{hc}{E_3 - E_1} = 103nm$$

H1.4 Calculate the kinetic and potential energy of electron in the ground state of Hydrogen atom.

Solution:

Potential energy in H atom,

$$V = \frac{-e^2}{4\pi\varepsilon_o r}$$

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$$= \frac{-me^4}{16\pi^2\varepsilon_0^2\hbar^2} \qquad \text{(r = Bohr radius)}$$

Substituting the constants,

$$V = -27.2eV$$

Kinetic energy, T = Total energy – Potential energy

$$= -13.6eV + 27.2eV$$

$$T = 13.6eV$$

H1.5 What is the lowest wavelength present in Hydrogen emission spectrum?

Solution:

In H spectrum, the lowest wavelength should correspond to highest energy transition.

Rydberg formula

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For highest energy transition, (Lyman Series)

$n_1 = 1$ $\qquad$ $n_2 = \infty$

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = R$$

$$\lambda = \frac{1}{R} = 912\text{Å}$$

H1.6 Evaluate the ratio of de Broglie wavelengths of electron and proton when they have the same kinetic energy.

Solution:

Kinetic energy

Electron $\qquad T = \frac{p^2}{2m}$

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$$\lambda_1 = \frac{h}{p_1} = \frac{h}{\sqrt{2m_1T_1}} \quad \text{-------- (1)} \qquad p = \sqrt{2mT}$$

Proton

$$\lambda_2 = \frac{h}{p_2} = \frac{h}{\sqrt{2m_2T_2}} \quad \text{-------- (2)}$$

(1)/(2),

$$\frac{\lambda_1}{\lambda_2} = \frac{h}{\sqrt{2m_1T_1}}\frac{\sqrt{2m_2T_2}}{h} = \frac{\sqrt{m_2T_2}}{\sqrt{m_1T_1}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}} \qquad \text{(since } T_1 = T_2\text{)}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{1836}{1}}$$

$$\frac{\lambda_1}{\lambda_2} = 42.8$$

H1.7 An electron accelerated by 5 kV potential is located within a distance of 3 nm in an experiment. Calculate the percentage uncertainty in the momentum of the electron.

Solution: An electron accelerated by 5kV potential has kinetic energy

$$T = \frac{p^2}{2m} = 5\text{keV}$$

Momentum, $p = \sqrt{2mT}$

$$p = 3.818 \times 10^{-23} \text{ kg m s}^{-1}$$

Position uncertainty, $\Delta x = 3 \times 10^{-9} m$

$$\Delta x \Delta p = \frac{h}{4\pi}$$

$$\Delta p = \frac{h}{4\pi\Delta x} = \frac{6.626\times10^{-34}}{4\times3.14\times3\times10^{-9}} = 0.1758 \times 10^{-25}$$

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$$\Delta p = 0.001758 \times 10^{-23} \text{kg m s}^{-1}$$

Percentage Uncertainty = $\frac{\Delta p}{p} \times 100$ =4.605×10^{-2} %

H1.8 The wavefunction of a particle of mass m moving in a potential, $V(x) = \alpha^2 x^2$ is given by $\psi(x) = e^{-\sqrt{m\alpha^2/2\hbar^2}x^2}$, where α is a constant. Find the energy of the particle.

Solution:
$$\varphi(x) = e^{-\sqrt{\frac{\pi m\alpha^2}{2\hbar^2}}x^2}$$

$$\frac{d\varphi(x)}{dx} = -\sqrt{\frac{2m\alpha^2}{\hbar^2}}xe^{-\sqrt{\frac{m\alpha^2}{2\hbar^2}}x^2}$$

$$\frac{d^2\varphi(x)}{dx^2} = -\sqrt{\frac{2m\alpha^2}{\hbar^2}}(1 - \sqrt{\frac{2m\alpha^2}{\hbar^2}}x^2)e^{-\sqrt{\frac{m\alpha^2}{2\hbar^2}}x^2}$$

Substitute above in TISE

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi(x)}{dx^2} + V(x) = E\varphi(x)$$

$$-\frac{\hbar^2}{2m}\left(-\sqrt{\frac{2m\alpha^2}{\hbar^2}}\left(1 - \sqrt{\frac{2m\alpha^2}{\hbar^2}}x^2\right)\right)e^{-\sqrt{\frac{m\alpha^2}{2\hbar^2}}x^2} + V(x)e^{-\sqrt{\frac{m\alpha^2}{2\hbar^2}}x^2} = Ee^{-\sqrt{\frac{m\alpha^2}{2\hbar^2}}x^2}$$

$$E = \frac{\hbar\alpha}{\sqrt{2m}}$$

H1.9 Consider an electron microscope which can apply a maximum of 40 kV potential to accelerate electrons. Using this microscope, what will be the smallest entity (let's call it microbe X) that can be seen? If you were to use neutrons instead of electrons in the machine, what must the energy of neutrons in order to look at microbe X?

Solution: Maximum KE of electrons

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T=40 keV $=\frac{p^2}{2m}$

$$p^2 = 2 \times 9.1 \times 10^{-31} \times 40 \times 10^3 \times 1.6 \times 10^{-19} = 116.48 \times 10^{-46}$$

$p = 10.793 \times 10^{-23}$kg m s^{-1}

de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}}{10.793\times10^{-23}} = 0.6139 \times 10^{-11}$

λ=6.139 pm

An entity of size $\sim$ 6.139 pm can be seen using the microscope.

If we want to use neutrons, they should have de Broglie wavelength of 6.139pm

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}}{6.139\times10^{-12}}$$

$p = 1.079 \times 10^{-22}$kg m s^{-1}

$$T = \frac{p^2}{2m} = \frac{(1.079\times10^{-22})^2}{2\times1836\times9.1\times10^{-31}} = 21.8 \text{ eV}$$

H1.10 Consider a particle of mass m constrained to move on a circle of radius a. The time independent Schrödinger equation for this system can be written as

$$\frac{-\hbar^2}{2I}\frac{d^2(\Psi(\theta))}{d\theta^2} = E\Psi(\theta); \qquad 0 \le \theta \le 2\pi \qquad (1)$$

where $I = ma^2$ is the moment of Inertia and θ is the angle that describes the position of the particle on the ring.

(a) By substitution, show that the solutions to this equation are $\Psi(\theta) = Ae^{i\alpha\theta}$ where $\alpha = \pm\sqrt{2IE}/\hbar$.

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(b) **Argue that the appropriate boundary condition for this problem is $\boldsymbol{\Psi(\theta) = \Psi(\theta + 2\pi)}$.**

(c) **Use the condition in (b) to find an expression for energy E.**

(d) **Find out the normalization constant A.**

Solution:

(a) Substituting $\varphi(\vartheta) = Ae^{i\alpha\vartheta}$ in equation (1)

$$-\frac{\hbar^2}{2I} Ae^{i\alpha\vartheta}(i\alpha)^2 = EAe^{i\alpha\vartheta}$$

$$e^{i\alpha\vartheta} = e^{i\alpha\vartheta}$$

(b) Since the particle moves on a circle of radius a, $\varphi(\vartheta)$ and $\varphi(\vartheta + 2\pi)$ are not unique solutions to the schrodinger's equation. Hence, setting $\varphi(\vartheta) = \varphi(\vartheta + 2\pi)$ is appropriate.

(c) Using the boundary condition ,

$$\varphi(\vartheta) = \varphi(\vartheta + 2\pi)$$

$$Ae^{i\alpha\vartheta} = Ae^{i\alpha(\vartheta+2\pi)}$$

$$e^{2\pi i\alpha} = 1$$

α=n, n=0,$\pm$1, $\pm$2, … …

$$n = \frac{\pm\sqrt{2IE}}{\hbar}$$

$$E = \frac{n^2\hbar^2}{2I}$$

(d)

$$\int_0^{2\pi} \Psi^*(\theta)\Psi(\theta)d\theta = 1$$

$$A^2 \int_0^{2\pi} e^{-i\alpha\theta} e^{i\alpha\theta} d\theta = 1$$

$$A = \frac{1}{\sqrt{2\pi}}$$