## Homework 2

**Instructions**

(1) Write the answers clearly and show the necessary mathematics.
(2) Doing homeworks is essential to understand the subject.
(3) For your own good, do not copy.
(4) Submission due date: 17 September 2014
(5) Assignment problems are not an exhaustive list of problems. You are encouraged to do more problems presented in standard books.

H2.1 Consider a particle confined to a 2D box of dimensions $(0 \le x \le a; 0 \le y \le b)$. Suppose the particle is described by the state function $\psi(x,y) = \frac{30}{\sqrt{a^5 b^5}} x(a-x)(b-y)$. Find an expression for $\langle E \rangle$ in terms of the box dimensions.

H2.2 Consider a particle confined to a 3D box of dimensions $(0 \le x \le a; 0 \le y \le b; 0 \le z \le c)$. For this system,

(a) find $\langle x \rangle$, $\langle y \rangle$, and $\langle z \rangle$
(b) find $\langle p_x \rangle$
(c) is $\langle x^2 \rangle = \langle x \rangle^2$?
(d) is $\langle xy \rangle = \langle x \rangle \langle y \rangle$?

H2.3 Consider a one particle, one dimensional system described by $\psi(x) = A\left(\frac{x}{x_0}\right)^n e^{-\frac{x}{x_0}}$, where $A, x_0$ and $n$ are constants. Under the condition, $V(x) \to 0$ when $x \to \infty$, find an expression for the energy of the system.

H2.4 Consider the one dimensional quantum harmonic oscillator. Suppose the particle is described by the function $\emptyset(x) = c_0\psi_0 + c_1\psi_1$, where $c_0$ and $c_1$ are real constants and $\psi_0$ and $\psi_1$ are ground state and first excited state harmonic oscillator wavefunctions. Compute $\langle x \rangle$ in terms of $c_0$ and $c_1$.

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H2.5 A particle of mass $m$ moves in a one dimensional space under the influence of a potential $V(x)$. The state of the particle is described by $\psi(x) = \left(\frac{\alpha^2}{\pi}\right)^{1/4} e^{-\frac{\alpha^2 x^2}{2}}$ with energy $E = \frac{\hbar^2 \alpha^2}{2m}$. Find $\langle x \rangle$, $\langle p_x \rangle$ and $V(x)$.

H2.6 Find $\langle x \rangle$ and $\langle p_x \rangle$ for the $v^{\text{th}}$ state of one dimensional quantum harmonic oscillator.

H2.7 The strongest vibrational absorption band of $^{12}C^{16}O$ molecule occurs at $6.43 \times 10^{13}$ Hz. If the reduced mass of $^{12}C^{16}O$ is $1.385 \times 10^{-26}$ kg, calculate the zero point energy and bond strength of the $^{12}C^{16}O$ bond. State the approximation you made in the calculation.

H2.8 A particle of mass $m$ confined to move in a potential $V(x) = 0$ for $0 \leq x \leq a$ and $V(x) = \infty$ otherwise. The wavefunction of the particle at time $t = 0$ is given by

$$\psi(x, 0) = A \sin\left(\frac{5\pi x}{a}\right) \cos\left(\frac{2\pi x}{a}\right)$$

(a) Normalize $\psi(x, 0)$.
(b) Find $\psi(x, t)$.

H2.9 Consider a particle moving in the potential $V(x) = \frac{1}{2}kx^2 + cx$, where $c$ is a constant. Find the energy eigenvalues for this particle.