21002: Quantum Mechanics and Applications

2014-15 First Semester (July – November 2014)

Homework 4

Instructions

(1) Write the answers clearly and show the necessary mathematics.

(2) Doing homeworks is essential to understand the subject.

(3) For your own good, do not copy.

(4) Submission due date: 08 October 2014

(5) Assignment problems are not an exhaustive list of problems. You are encouraged to do more problems presented in standard books.

H4.1 Consider two arbitrary kets $\{|\alpha>, |\beta>\}$. The corresponding wavefunctions in position and momentum representations are $\{\psi_\alpha(x), \psi_\beta(x)\}$ and $\{\phi_\alpha(p), \phi_\beta(p)\}$, respectively.

(a) How will you obtain $\psi_\alpha(x)$ from $|\alpha>$ and $\phi_\beta(p)$ from $|\beta>$?

(b) Given that operator $\widehat{f(x)}$ is a function of x, obtain an expression for the matrix element $<\beta|f(x)|\alpha>$ in terms of $\psi_\alpha(x)$ and $\psi_\beta(x)$.

H4.2 In the two state Stern-Gerlach (z) system, the basekets are $\{|S_z^+>, |S_z^->\}$ and the operator involved is $\widehat{S_z}$. In this formalism, lets construct the following two operators.

$$\hat{S}_+ = \hbar|S_z^+><S_z^-|$$

$$\hat{S}_- = \hbar|S_z^-><S_z^+|$$

Find out the matrix representations of $\hat{S}_+$ and $\hat{S}_-$ in the $\{|S_z^+>, |S_z^->\}$ basis. Are these operators Hermitian?

H4.3 Consider the two observables S_x and S_y and their eigenkets $\{|S_x^+>, |S_x^->\}$ and $\{|S_y^+>, |S_y^->\}$, respectively. Construct the transformation matrix that changes the base from $\{|S_x^+>, |S_x^->\}$ to $\{|S_y^+>, |S_y^->\}$ and show that it's an unitary matrix.

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H4.4 For a given arbitrary ket $|\alpha>$, prove the following.

(a) $<p'|x|\alpha>= i\hbar \frac{\partial \phi_\alpha(p')}{\partial p'}$

(b) $<\beta|x|\alpha>= \int dp' \phi_\beta^*(p') i\hbar \frac{\partial}{\partial p'} \phi_\alpha(p')$

H4.5 Consider the physical property M and the corresponding Hermitian operator $\widehat{M}$. The Hilbert space of $\widehat{M}$ is spanned by the complete set $\{|m_1>, |m_2>, |m_3>\}$ with non-degenerate eigenvalues m_1, m_2, m_3, respectively. The system under consideration exists as a superposition state $|\alpha>$ which is given as

$$|\alpha>= c_1|m_1> + c_2|m_2> + c_3|m_3>$$

Where $c_1 = 0.447 + 0.316i, c_2 = 0.500 - 0.316i, c_3 = 0.387 + 0.448i$

(a) Verify that $|\alpha>$ is normalized.
(b) Write down the expression for $<\alpha|$.
(c) Write down the completeness relation.
(d) When you measure M, what is the probability for $M = m_2$?
(e) Consider another physical property N which satisfies the condition $\widehat{N}|m_i>= n_i|m_i>$, where $i = 1,2,3$ and n_1, n_2, n_3 are non-degenerate. Write down $\widehat{N}$ in terms of $|m_i>$.
(f) Do $\widehat{M}$ and $\widehat{N}$ commute?
(g) Write $|\alpha>$ and $<\alpha|$ in the $|m_i>$ basis.
(h) Find out the trace of $\widehat{N}$.
(i) Write down the wavefunction of $|\alpha>$ in the position and momentum representations.