## Homework 6

**Instructions**

(1) Write the answers clearly and show the necessary mathematics.
(2) Doing homeworks is essential to understand the subject.
(3) For your own good, do not copy.
(4) Submission due date: 14 November 2014
(5) Assignment problems are not an exhaustive list of problems. You are encouraged to do more problems presented in standard books.

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*Note*: Wherever required, consider the orthonormal basis $\{|0>, |1>\}$.

H6.1 Show that the following system is in a mixed state.

$$\left|\psi_1 >= \frac{2}{\sqrt{5}}\right| 0 > + \frac{1}{\sqrt{5}}|1 > \qquad w_1 = 0.65$$

$$\left|\psi_2 >= \frac{\sqrt{3}}{2}\right| 0 > + \frac{1}{2}|1 > \qquad w_1 = 0.35$$

H6.2 Find out the system described by the following density operator is a pure state or not.

$$\rho = \frac{1}{4}|0><0| + \frac{3}{4}|1><1| + \frac{\sqrt{3}}{4}|0><1| + \frac{\sqrt{3}}{4}|1><0|$$

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H6.3 An ensemble $Q_1$ is prepared such that $|0>$ is occupied with probability ¼ and $|1>$ is occupied with probability ¾.

Let's define

$$|a>=\sqrt{\frac{1}{4}}\ |0>\ +\ \sqrt{\frac{3}{4}}\ |1>$$

$$|b>=\sqrt{\frac{1}{4}}\ |0>\ -\ \sqrt{\frac{3}{4}}\ |1>$$

Another ensemble $Q_2$ is prepared such that the probability to occupy $|a>$ is ½ and to occupy $|b>$ is ½. Show that the density operators for ensembles $Q_1$ and $Q_2$ are the same.

H6.4 For any arbitrary state kets $|A>$ and $|B>$ show that $Tr(|A><B|) =< B|A >$.

H6.5 Suppose a composite of systems $A$ and $B$ is in the state $|01>$, where $|0>$ is a pure state of system $A$ and $|1>$ is a pure state of system $B$. Find the reduced density operator of system $A$ and show it's a pure state.

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H6.6 Compute and compare the Von Neumann Entropy for the following two systems.

(a) $|\psi>=\frac{1}{\sqrt{2}}(|0>+|1>)$

(b) $|\psi_1>=|0>, \quad w_1=0.75$

$|\psi_2>=\frac{1}{\sqrt{2}}(|0>-|1>), \quad w_2=0.25$

H6.7 A two qubit system is in the following Bell state

$$|\phi>_{AB}=\frac{1}{\sqrt{2}}(|00>_{AB}+|11>_{AB})$$

Compute the Entropies, $S(\rho_{AB})$ and $S(\rho_A)$.