21002: Quantum Mechanics and Applications

2014-15 First Semester (July – November 2014)

Midsem I **04/09/2014** **Points: 20** **Time: 1 hr**

Instructions

(1) There are two pages in the question paper.

(2) Write the question number and answers clearly and show the necessary mathematics.

Good Luck!

1. An electron is confined by infinite potential barriers to move in a one dimensional box of length 2 nm.
   (a) For states ψ_1 and ψ_2, point out at what values of x the probability to locate the electron will be maximum and minimum. (*2 points*)
   (b) Suppose the electron is in state ψ_1. To excite the electron to ψ_3, what wavelength of light photon is needed? (*2 points*)
   (c) Assume there are two electrons in the box instead of one, and their repulsion potential is given by $e^2/4\pi\epsilon_0 x_{12}$, where x_{12} is the distance between electrons 1 and 2, respectively. Setup the Schrödinger equation for this system and indicate what will be your approach to solve it. (*2 points*)

2. Consider an electron in a three dimensional cubic box with wavefunctions Ψ_{n_x,n_y,n_z} and energy E_{n_x,n_y,n_z}. Indicate whether the following two functions are eigenfunctions of the Hamiltonian operator of the system. Justify your answer. (*4 points*)

$$\phi_I = \frac{1}{\sqrt{2}}\left(\Psi_{2,4,9} + \Psi_{9,2,4}\right)$$

$$\phi_{II} = \frac{1}{\sqrt{2}}\left(\Psi_{3,5,1} + \Psi_{1,5,1}\right)$$

3. You are accelerating a beam of 10000 electrons towards a potential barrier of height 6 eV and 0.2 nm wide. What should be your applied potential to accelerate the electrons if you want approximately 300 electrons to go through the barrier? (*5 points*)

4. Consider a particle in the ground state of one dimensional harmonic oscillator. The classical and quantum energy expressions are, (*5 points*)

$$E_{clas} = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$E_{quan} = \left(v + \frac{1}{2}\right)\hbar\omega$$

$$\omega = \sqrt{\frac{k}{m}}$$

The ground state wavefunction of the particle is $\Psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{\frac{-\alpha x^2}{2}}$, with $\alpha = \frac{\omega m}{\hbar}$. Calculate the total probability for the particle (in the ground state) to be found in the classically forbidden region. Assume that you are working in an arbitrary unit system in which $m = \omega = \hbar = 1$. (*Hint*: Compute the classical turning points.)

Useful data and integrals

Speed of light, $c = 3.0 \times 10^{8}$ ms^{-1}

Mass of electron, $m_e = 9.109 \times 10^{-31}$ kg

Charge on electron, $e = 1.602 \times 10^{-19}$ C

$\int_1^\infty e^{-z^2} dz = 0.1418$

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

1 eV = 1.602×10^{-19} J

$\int_0^\infty x^n\, e^{-qx} dx = \frac{n!}{q^{n+1}}$

$\int_0^\infty e^{-bx^2} dx = \frac{1}{2}\left(\frac{\pi}{b}\right)^{\frac{1}{2}}, \quad b > 0$