**21002: Quantum Mechanics and Applications**

2014-15 First Semester (July – November 2014)

**Midsem II** **09/10/2014** **Points: 20** **Time: 1 hr**

**Instructions**

(1) There are two pages in the question paper.
(2) Write the question number and answers clearly and show the necessary mathematics.

*Good Luck!*

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1. You are doing an experiment to measure the *z*-component of angular momentum of a particle. The particle is described by the function $Y_l^m$ which satisfies the eigenvalue equation $\hat{L}^2 Y_l^m = 6\hbar^2 Y_l^m$ ($Y_l^m$ are the spherical harmonics). What are the possible outcomes of your experiment. (2 points)

2. In class, we solved the harmonic oscillator problem in the Schrödinger formalism. In matrix mechanics, one solves the same problem in the "number basis". In this formalism, the energy eigenvalue equation for harmonic oscillator is given as (4 points)

$$\hat{H}|n> = \left(n+\frac{1}{2}\right)h\nu \ |n>$$

where $|n>$ are the energy eigenkets and $\left(n+\frac{1}{2}\right)h\nu$ are the eigenvalues with *n* = 0,1,2,… In this formalism, two non-Hermitian operators called annihilation operator ($\hat{a}$) and creation operator $\left(\hat{a}^\dagger\right)$ exist. Their action on $|n>$ is given below

$$\hat{a}|n> = \sqrt{n} \ |n-1>$$

$$\hat{a}^\dagger|n> = \sqrt{n+1} \ |n+1>$$

Assuming that Hilbert space of a given harmonic oscillator is completely described by the three energy eigenkets $\{|1>, |2>, |3>\}$, show the matrix representations of $\hat{a}$ and $\hat{a}^\dagger$ in the given basis.

3. Consider an electron confined to a one-dimensional box of length 1 *nm* with infinite potential walls. You are doing three experiments consecutively on the system to measure energy (*E*), momentum (*p*) and again energy (*E*). You observe 0.5 eV for *E* in the first experiment and $3.8 \times 10^{-25}$ $kgms^{-1}$ for *p* in the second experiment. What value of *E* you will observe in the third experiment? Explain your answer. (Assume no external disturbance during your measurements). (4 points)

4. Two arbitrary kets $|\alpha >$ and $|\beta >$ in <u>*A basis*</u> are (4 points)

$$|\alpha > \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\beta > \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The same kets in <u>*B basis*</u> are

$$|\alpha > \doteq \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad |\beta > \doteq \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Construct a unitary matrix $U$ such that "<u>*B basis*</u> = $U^\dagger$ <u>*A basis*</u>" and prove that $U$ is unitary.

5. Hydrogen atom energy eigenfunctions and eigenvalues are given by

$$\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)Y_l^m(\theta,\phi)$$

$$E_n = \frac{-13.6\, Z^2}{n^2}\, eV$$

At a given time instant, the Hydrogen atom exists in a superposition state described by

$$\phi_{nlm}(r,\theta,\phi) = \frac{1}{\sqrt{10}}\left(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1}\right)$$

(a) Calculate the expectation value of energy $< E >_\phi$. (3 points)

(b) What is the probability for the system to have $l = 1$ and $m = +1$? (3 points)

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**Useful data and integrals**

Speed of light, $c = 3.0 \times 10^8$ ms$^{-1}$

Mass of electron, $m_e = 9.109 \times 10^{-31}$ kg

Charge on electron, $e = 1.602 \times 10^{-19}$ C

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

1 eV = $1.602 \times 10^{-19}$ J

$$\int_0^\infty e^{-bx^2} dx = \frac{1}{2}\left(\frac{\pi}{b}\right)^{\frac{1}{2}}, \quad b > 0$$