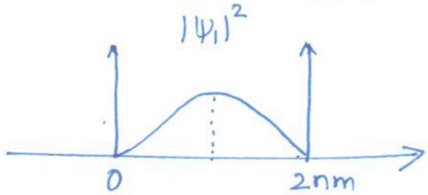
Midsem I Solutions

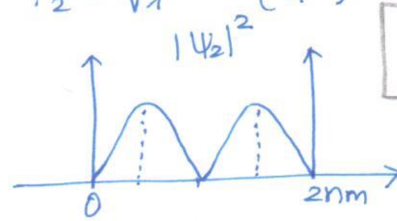
1

(a)

$$\psi_1 = \sqrt{\frac{2}{l}} \sin\left(\frac{\pi x}{l}\right) \qquad \psi_2 = \sqrt{\frac{2}{l}} \sin\left(\frac{2\pi x}{l}\right)$$

2 points





Maximum at 1nm.

Maxima at 0.5nm and 1.5nm
Minima at 1nm

(b)

$$\Delta E = h\nu = E_3 - E_1 = \frac{9h^2}{8ml^2} - \frac{h^2}{8ml^2} = \frac{h^2}{\cancel{8}ml^2}$$

$$\frac{hc}{\lambda} = \frac{h^2}{ml^2}$$

2 points

$$\lambda = \frac{c m l^2}{h}$$

$$= \frac{3\times10^{8}\ ms^{-1} \times 9.109\times10^{-31}\ kg \times (2\times10^{-9} m)^2}{6.626\times10^{-34}\ Js}$$

$$\boxed{\lambda = 1649\ nm}$$

(c) Schrodinger equation

$$\hat{H}\,\psi(x_1, x_2) = E\,\psi(x_1, x_2)$$

2 points

$x_1 \rightarrow$ coordinate of particle 1

$x_2 \rightarrow$ Coordinate of particle 2

$$\hat{H} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) + \frac{e^2}{4\pi\varepsilon_0 x_{12}}$$

The $\frac{e^2}{4\pi\varepsilon_0 x_{12}}$ term in the Hamiltonian makes the problem non-separable.

Hence, one has to use approximation methods to solve this problem.

(2) $\Psi_{2,4,9}$ and $\Psi_{9,2,4}$ are degenerate states of the system

$$E_{2,4,9} = \frac{h^2}{8ma^2}(4+16+81) = \frac{101\,h^2}{8ma^2}$$

$$E_{9,2,4} = \frac{h^2}{8ma^2}(81+4+16 = \frac{101\,h^2}{8ma^2}$$

Since Φ_1 is a linear combination of $E_{2,4,9}$ and $E_{9,2,4}$, it should also be an eigenfunction of the Hamiltonian.

$$H\,\Phi_1 = \hat{H}\frac{1}{\sqrt{2}}(\Psi_{2,4,9} + \Psi_{9,2,4})$$

$$= \frac{1}{\sqrt{2}}(\hat{H}\Psi_{2,4,9} + \hat{H}\Psi_{9,2,4})$$

$$= E_{2,4,9}\frac{1}{\sqrt{2}}(\Psi_{2,4,9} + \Psi_{9,2,4})$$

4 points

$$\boxed{H\,\Phi_1 = \underset{E_{9,2,4}}{\overset{E_{2,4,9}}{\text{(or)}}}\;\Phi_1}$$

$\Psi_{3,5,1}$ and $\Psi_{1,5,1}$ are not degenerate.

$$E_{3,5,1} = \frac{h^2}{8ma^2}(9+25+1) = \frac{35h^2}{8ma^2}$$

$$E_{1,5,1} = \frac{h^2}{8ma^2}(1+25+1) = \frac{27h^2}{8ma^2}$$

Hence, Φ_{II} is not an Eigenfunction of the Hamiltonian.

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(3)

$$T = e^{-2kL}$$

$$k = \frac{\sqrt{2m(U-E)}}{\hbar}$$

5 points

$$L = 0.2\times10^{-9}\,m$$

$$T = \frac{300}{10000} = 0.03$$

$$m = 9.109\times10^{-31}\,kg$$

-19

$$T = e^{-2kL}$$

$$\ln T = -2kL$$

$$k = \frac{-1}{2L} \ln T$$

$$= \frac{-1}{2 \times 0.2 \times 10^{-9} m} \ln[0.03]$$

$$k = 8.7664 \times 10^{9} m^{-1}$$

$$\sqrt{2m(U-E)} = \frac{kh}{2\pi}$$

$$E = U - \frac{k^2 h^2}{8 m \pi^2}$$

$$= 6 \times 1.602 \times 10^{-19} J - \frac{(8.7664 \times 10^{9} m^{-1})^2 \times (6.626 \times 10^{-34} Js)^2}{8 \times 9.109 \times 10^{-31} kg \times (3.14)^2}$$

$$= 4.91603 \times 10^{-19} J$$

$$\boxed{E = 3.069 eV}$$

we have to apply approximately 3V potential to accelerate the electrons.

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4 we are considering the ground state of Harmonic oscillator.

Hence $E = \frac{1}{2}\hbar\omega$

At the classical turning points, the total energy of the system is potential energy. Hence

$$\frac{1}{2}\hbar\omega = \frac{1}{2}kx^2$$

5 points

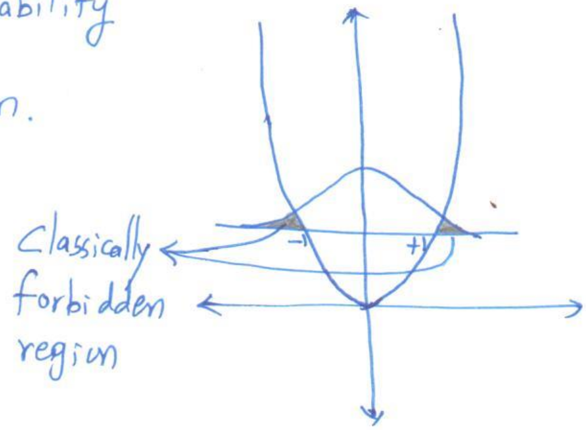
$$\hbar\omega = m\omega^2 x^2$$

In the given unit system $\hbar = m = \omega = 1$.

$$\boxed{x = \pm 1}$$

$x = \pm 1$ are the classical turning points for the given system in the given units.

We have to calculate the total probability in the classically forbidden region.



$$\text{Probability} = \int_{-1}^{-\infty} \psi^* \psi \, dx + \int_{1}^{\infty} \psi^* \psi \, dx$$

$$= 2 \int_{1}^{\infty} \psi^* \psi \, dx$$

$$= 2 \int_{1}^{\infty} \left(\frac{\alpha}{\pi}\right)^{\frac{1}{2}} e^{-\alpha x^2} \, dx$$

$$= \frac{2}{\sqrt{\pi}} \int_{1}^{\infty} e^{-x^2} \, dx \quad \left[\alpha = \frac{\omega m}{\hbar} = 1 \right]$$

$$= \frac{2}{\sqrt{\pi}} \times 0.1418$$

$$\underline{\underline{\text{Probability} = 0.16}}$$