Quiz # 2 Solutions

1 (a)
$$[\hat{x}, \hat{p}_x] f(x,y,z) = \frac{\hbar}{i}\left[x\frac{\partial f}{\partial x} - \frac{\partial}{\partial x} x f\right]$$
$$= \frac{\hbar}{i}\left[x\frac{\partial f}{\partial x} - f - x\frac{\partial f}{\partial x}\right]$$

$$[\hat{x}, \hat{p}_x] f(x,y,z) = -\frac{\hbar}{i} f \neq 0$$

$\therefore$ x and p_x cannot be simultaneously measured.

(b)
$$[\hat{x}, \hat{p}_y] f(x,y,z) = \frac{\hbar}{i}\left[x\frac{\partial f}{\partial y} - \frac{\partial}{\partial y} x f\right]$$
$$= \frac{\hbar}{i}\left[x\frac{\partial f}{\partial y} - \underbrace{\frac{\partial x}{\partial y}}_{0} f - x\frac{\partial f}{\partial y}\right]$$

$$[\hat{x}, \hat{p}_y] f(x,y,z) = 0$$

$\therefore$ x and p_y can be simultaneously measured.

2
$$\langle p \rangle = \int \psi^* \hat{p}\, \psi \, d\tau$$

Here, $\psi = \sqrt{\frac{2}{l}} \sin\left(\frac{\pi x}{l}\right)$

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$\langle p \rangle = \frac{2\hbar}{l i}\int_0^l \sin\left(\frac{\pi x}{l}\right)\frac{\partial}{\partial x}\sin\left(\frac{\pi x}{l}\right) dx$$

$$= \frac{2\hbar\pi}{l^2 i}\int_0^l \sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi x}{l}\right) dx$$

$$= \frac{\hbar\pi}{l^2 i}\int_0^l \sin\left(\frac{2\pi x}{l}\right) dx \qquad [\sin 2x = 2\sin x\cos x]$$

$$= \frac{\hbar\pi}{l^2 i}\left[\cos\left(\frac{2\pi x}{l}\right)\frac{l}{2\pi}\right]_0^l$$

$$= \frac{\hbar\pi l}{2\pi l^2 i}\left[\cos\left(\frac{2\pi l}{l}\right) - \cos 0\right]$$

$$\langle p \rangle = 0$$

3 $\hat{A}$ is Hermitian. Hence,

$$\int \psi^* (\hat{A}\psi)\, d\tau = \int \psi (\hat{A}\psi)^* d\tau \longrightarrow 1$$

a

$$\int \psi^* (c\hat{A}\psi)\, d\tau = \int \psi (c\hat{A}\psi)^* d\tau \longrightarrow 2$$

$$c\int \psi^* \hat{A}\psi\, d\tau = \int \psi c^* (\hat{A}\psi)^* d\tau$$

Using 1

$$c - c^* = 0$$

$$\boxed{c = c^*} \quad [c \text{ is real}]$$

Hence, 2 is obeyed.

b Given c is imaginary

$$\int \psi^* (c\hat{A}\psi)\, d\tau = \int \psi (c\hat{A}\psi)^* d\tau \longrightarrow 3$$

$$c\int \psi^* \hat{A}\psi\, d\tau = -c^* \int \psi (\hat{A}\psi)^* d\tau$$

$$c = -c^*$$

[For a purely imaginary number a, $a = -a^*$]

3 is not obeyed.