Quiz #3 Solutions

① (a) FALSE

If we have n non-interacting particles, their total wavefunction is written as

$$\psi = \psi_1(q_1)\,\psi_2(q_2)\cdots\psi_n(q_n) = \prod_{i=1}^{n}\psi_i(q_i)$$

(b) TRUE

The energy of a system of non-interacting particles is the sum of the energies of the individual particles.

$$E = E_1 + E_2 + \cdots + E_n = \sum_{i=1}^{n} E_i$$

(c) FALSE

For rigid rotor system,

$$H\psi = E\psi$$

$$E = \frac{J(J+1)}{2I}, \quad J = 0, 1, 2, \ldots$$

when $J=0$, $E=0$ which is allowed.

(d) TRUE

$$\langle B \rangle = \int \psi^* \hat{B} \psi \; r^2 \sin\theta \, dr\,d\theta\,d\phi$$

$r^2\sin\theta$ is the part of volume element and $\hat{B}$ should not act on it.

2

Consider

$$\hat{H}\,\psi_n = E_n\,\psi_n$$

For $\hat{H}$ to be Hermitian,

$$\int \psi_n^*\,\hat{H}\,\psi_n\,d\tau = \int \psi_n\,(\hat{H}\,\psi_n)^*\,d\tau$$

$$\int \psi_n^*\,E_n\,\psi_n\,d\tau = \int \psi_n\,E_n^*\,\psi_n^*\,d\tau$$

$$(E_n - E_n^*)\int \psi_n^*\,\psi_n\,d\tau = 0$$

$$\boxed{E_n = E_n^*}$$

Hence Eigenvalues of a Hermitian operator are real.