Quiz #4 Solutions

①

Time dilation

$$\Delta t = \frac{\Delta t_p}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \Delta t_p \rightarrow \text{Proper time}$$

$\Delta t_p = 3600\,s$

$\Delta t = 3601\,s$

$c = 3\times10^8\,ms^{-1}$

$$\Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{\Delta t_p}{\Delta t}$$

$$1-\frac{v^2}{c^2} = \left(\frac{\Delta t_p}{\Delta t}\right)^2$$

$$\frac{v^2}{c^2} = 1-\left(\frac{\Delta t_p}{\Delta t}\right)^2$$

$$v = c\sqrt{1-\left(\frac{\Delta t_p}{\Delta t}\right)^2} = 3\times10^8\,ms^{-1}\sqrt{1-\left(\frac{3600}{3601}\right)^2} = 0.071\times10^8\,ms^{-1}$$

$$\boxed{v = 7.1\times10^6\,ms^{-1}}$$

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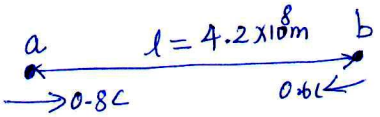
②

$t = 0$



$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{4.2\times10^8\,m}{1.4\,c} = \frac{4.2\times10^8\,m}{1.4\times3\times10^8\,ms^{-1}} = 1s$$

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③

$$\Delta t = \frac{\Delta t_p}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\Delta t_p = 3s$

$v = 0.95c$

$$\Delta t = \frac{3}{\sqrt{1-\frac{(0.95c)^2}{c^2}}} = \frac{3}{\sqrt{1-0.95^2}}$$

$$\boxed{\Delta t = 9.6s}$$

(4) The answer is no. In the frame of the spacecraft, the length will not change.

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(5) Length contraction

$$L = L_p \sqrt{1 - \frac{v^2}{c^2}}$$

$$= 125\,m \sqrt{1 - \frac{(0.99c)^2}{c^2}}$$

$$\boxed{L = 17.6\,m}$$
