Tutorial 2

T2.1 In class, you learned about Bohr's theory of Hydrogen atom. Before Bohr, Rutherford tried to explain hydrogen atomic structure by assuming a planetary model (which you are acquainted from your high school). Rutherford's model is a classical model and it predicts that light emitted by hydrogen atom is equal to the frequency of the moving electron in the orbits. According to Bohr, the frequency of light emitted is a result of electron making jumps between stationary states i.e. $\upsilon = (E_{n1} - E_{n2})/h$. At highvalues of n, show that Bohr and Rutherford models converge. This convergence is an example of a rule called "correspondence principle" which states that in the limit of large quantum numbers, quantum mechanics recovers classical mechanics.

Bohr Model of atom

Coulomb force = Centrifugal force

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \qquad \text{----------- (1)}$$

where r = radius of orbit

v = speed of electron

m= mass of electron

Bohr's Condition

$$mvr = n\hbar$$

$$v = \frac{n\hbar}{mr} \qquad \text{---------- (2)}$$

from (1) and (2),

speed, $$v = \frac{nh}{2\pi mr} \qquad \text{--------- (3)}$$

$$r = \frac{\epsilon_0 n^2 h^2}{m\pi e^2} \qquad \text{--------- (4)}$$

$$E = \frac{-me^4}{8\epsilon_0^2 h^2 n^2} \qquad \text{--------- (5)}$$

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Bohr frequency

$h\upsilon = \Delta E = E_{n2} - E_{n1}$

$$h\upsilon = \frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\upsilon = \frac{me^4}{8\epsilon_0^2 h^3}\left(\frac{n_2^2 - n_1^2}{n_1^2 n_2^2}\right)$$

$$\upsilon = \frac{me^4}{8\epsilon_0^2 h^3}\frac{(n_2 + n_1)(n_2 - n_1)}{n_1^2 n_2^2}$$

Since n_1 and n_2 are large

$$\upsilon = \frac{me^4}{8\epsilon_0^2 h^3}\frac{2n_1}{n_1^4}$$

$$\upsilon = \frac{me^4}{4\epsilon_0^2 h^3 n_1^3} \qquad \text{----------} (6)$$

According to Rutherford, frequency of light by H atom is equal to the frequency of electron in the orbit.

Angular frequency of the electron

$$W_n = \frac{v_n}{r_n} \qquad \text{-----------} (7)$$

v_n = speed of electron in n^{th} orbit

r_n = radius of n^{th} orbit

Substitute (3) and (4) in (7),

$$W_n = \frac{nh}{2\pi m r^2} = \frac{nh}{2\pi m}\frac{m^2\pi^2 e^4}{\epsilon_0^2 n^4 h^4}$$

$$W_n = \frac{m\pi e^4}{2\epsilon_0^2 n^3 h^3} \qquad \text{-----------} (8)$$

(6) and (8) are approximately same. At large values of 'n', the Bohr's frequency and Rutherford's frequency are almost the same.

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T2.2 Somewhere in the universe, you are living in a place called "quantum land" where Planck's constant value is h = 10^{-1} Js. In that land, watermelons grow to a size of 20 cm with very hard shells and with seeds of mass 1.0 g. When you live in that land, and planning to eat watermelon, you have to be very careful while cutting open the melon. Explain why?

<u>Uncertainty Principle</u>

$$\Delta x \Delta p = \frac{h}{4\pi}$$

$$\Delta p = \frac{h}{4\pi \Delta x} = \frac{0.1}{4*3.14*0.2} = 3.98 * \ 10^{-2} \, kgms^{-1}$$

$$\Delta p = m\Delta v$$

$$\Delta v = \frac{\Delta p}{m} = \frac{3.98 * 10^{-2}}{10^{-3}} = 39.8 \, ms^{-1}$$

So the melon seeds will travel at a speed of ~40ms^{-1} when you cut open it. So you need to be careful.

T2.3 At a certain instant of time, a one-particle, one-dimensional system has

$$\Psi = (2/b^3)^{1/2} x e^{-|x|/b}, \text{ where } b = 3 \text{ nm}.$$

If a measurement of x is made at this time instant,

(a) find the probability for the particle to lie between 0.9 nm and 0.9001 nm (this interval can be treated as infinitesimal)

(b) find the probability for the particle to lie between 0 and 2 nm

(c) for what value of x, the probability density is a minimum?

(d) verify that $\psi$is normalized.

(a) Probability = $|\psi|^2 \, dx = \frac{2}{b^3} x^2 e^{-\frac{2|x|}{b}} dx$

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Here dx=0.0001 nm

$$|\psi|^2\, dx = \frac{2}{(3\times10^{-9})^3}(0.9\times10^{-9})^2 e^{-\frac{2(0.9\times10^{-9})}{3\times10^{-9}}}(0.0001\times10^{-9})$$

$|\psi|^2\, dx$= 3.29×10^{-6}

(b) For x>0, $|x|$=x

Probability= $\int_0^2 |\psi|^2 dx$

$= \frac{2}{b^3}\int_0^2 x^2 e^{-\frac{2x}{b}}dx$

$=\frac{2}{b^3}\left[e^{-\frac{2x}{b}}\left[-\frac{bx^2}{2}-\frac{b^2x}{2}-\frac{b^3}{4}\right]\right]_0^2$ (Refer table of integrals)

=0.0753

(c) ψ is zero at x = 0,

Hence $|\psi|^2$=0 at x=0 is the minimum possible probability density

(d) <u>Normalization condition</u>

$$\int_{-\infty}^{\infty} |\psi|^2\, dx = 1$$

$\int_{-\infty}^{\infty} |\psi|^2\, dx = \frac{2}{b^3}\int_{-\infty}^{0} x^2 e^{\frac{2x}{b}}dx + \frac{2}{b^3}\int_0^{\infty} x^2 e^{-\frac{2x}{b}}dx$ ----------(1)

Let w = -x in the final integral of RHS of (1)

$\int_{-\infty}^{0} x^2 e^{\frac{2x}{b}}dx = \int_{\infty}^{0} w^2 e^{-\frac{2w}{b}}(-dw)$

$= \int_0^{\infty} w^2 e^{-\frac{2w}{b}}dw$

This is equal to the second part of integral of RHS of (1)

Hence, $\int_{-\infty}^{\infty} |\psi|^2\, dx = \frac{4}{b^3}\int_0^{\infty} x^2 e^{\frac{-2x}{b}}dx$

$= \frac{4}{b^3}\left[\frac{2!}{(b/2)^3}\right] = 1$