Tutorial 3

T3.1 In class, we saw the energy expression for a particle confined to a potential well with $E < V_o$, where E and V_o are the total energy of the particle and potential barrier, respectively.

$$(2E - V_o)\sin\left[(2mE)^{1/2}l/\hbar\right] = 2(V_oE - E^2)^{1/2}\cos\left[(2mE)^{1/2}l/\hbar\right]$$

The wavefunctions are

$$\Psi_I = Ce^{\left[(2m/\hbar^2)^{1/2}(V_o-E)^{1/2}x\right]}$$

$$\Psi_{II} = A\cos\left[(2m/\hbar^2)^{1/2}E^{1/2}x\right] + B\sin\left[(2m/\hbar^2)^{1/2}E^{1/2}x\right]$$

$$\Psi_{III} = Ge^{\left[-(2m/\hbar^2)^{1/2}(V_o-E)^{1/2}x\right]}$$

Where *A*, *B*, *C* and *G* are constants (refer your notes). What happens to these energy and wavefunctions when $V_o \to \infty$ (i.e. when the finite barrier becomes an infinite barrier).

T3.2 For the particle in a 3D box system, when the sides of the box are same $(a = b = c)$, degeneracy is introduced in the system. Is it possible to see degeneracy even when $a \neq b \neq c$?

T3.3 Find the recursive relation for the differential equation given below and express the coefficient c_4 in terms of c_0 and c_5 in terms of c_1.

$$(1 - x^2)y''(x) - 2xy'(x) + 3y(x) = 0$$

Tutorial 3

T3.4 In class, we solved the one dimensional harmonic oscillator problem. Consider a three dimensional harmonic oscillator whose classical Hamiltonian is given by

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{k_x x^2}{2} + \frac{k_y y^2}{2} + \frac{k_z z^2}{2}$$

Setup the Schrödinger equation and find the energy eigenfunctions and eigenvalues. For this system, under what circumstances the energy levels will become degenerate? Suppose for the above system, the Hamiltonian is given by

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{k_x x^2}{2} + \frac{k_y y^2}{2} + \frac{k_z z^2}{2} + xy^2\sqrt{z}$$

Then how will you solve the Schrödinger equation?