Tutorial 5 Solutions

(1)

(a) **YES**

This is a 3 dimensional problem with V independent of θ and ϕ.

Radial equation

$$-\frac{\hbar^2}{2m}\left(R'' + \frac{2}{r}R'\right) + \frac{l(l+1)\hbar^2}{2mr^2}R = ER$$

(b) **YES**

$$V = \frac{k_x x^2}{2} + \frac{k_y y^2}{2} + \frac{k_z z^2}{2}$$

$$= \frac{k_x}{2}(x^2+y^2+z^2) \quad [\text{Given } k_x = k_y = k_z]$$

$$V = \frac{k_x}{2} r^2 \quad [r^2 = x^2+y^2+z^2]$$

V is a function of r only.

$$-\frac{\hbar^2}{2m}\left(R'' + \frac{2}{r}R'\right) + \frac{l(l+1)\hbar^2}{2mr^2}R + \frac{k_x}{2}r^2 R = ER$$

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(2) $r =$ constant is called the rigid rotor problem.

$$\hat{H} = \frac{-\hbar^2}{2\mu}\nabla^2$$

Since r is a constant, terms involving derivatives of r are omitted.

$$\Longrightarrow \hat{H} = \frac{\hat{L}^2}{2\mu d^2} \qquad d \to \text{distance between the 2 particles}$$

$$\hat{H}\psi = E\psi$$

$$\frac{\hat{L}^2}{2\mu d^2} R(r)\, Y_l^m(\theta,\phi) = E\, R(r)\, Y_l^m(\theta,\phi)$$

$$\frac{l(l+1)\hbar^2}{\ldots} Y_l^m(\theta,\phi) = E\, Y_l^m(\theta,\phi)$$

$$E = \frac{J(J+1)\hbar^2}{2\mu d^2}$$

$J = 0, 1, 2, \ldots$

[It is a convention to use J instead of l in rigid rotor problem]

The rotational motion of a diatomic molecule can be modeled using rigid rotor. Here μ is the reduced mass of the diatomic molecule and d is the bond distance. The approximation we make here is that the molecule doesn't vibrate i.e., d is a constant.

when $J = 0$, $E = 0$. Does this violate uncertainty principle?

NO

when $J = 0$, angular momentum is zero and hence $\Delta L = 0$. But we do not have angle information because there is no potential energy in the system.

But in case of harmonic oscillator $E = 0$ is not allowed because $E = 0$ would mean that $\Delta p = 0$ and $\Delta x = 0$.

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(3) In particle in a infinite well, the particle never escapes the well and it is always bound. Similarly in harmonic oscillator, the particle is bound to $V = \frac{1}{2}kx^2$ till $x \to \infty$. But in case of hydrogen atom the particle is bound to the nucleus only for some value of r and then it essentially becomes a free particle (ionization).
