21002: Quantum Mechanics and Applications

2014-15 First Semester (July – November 2014)

Tutorial 6

T6.1 Consider a physical property A and the corresponding Hermitian operator $\hat{A}$. The Hilbert space of $\hat{A}$ is spanned by the three orthonormal eigenkets $\{|a_1>, |a_2>, |a_3>\}$ with nondegenerate eigenvalues a_1, a_2 and a_3, respectively. Now a ket $|\alpha>$ is represented in this basis as

$$|\alpha>= c_1|a_1> + c_2|a_2> + c_3|a_3>$$

Where $c_1 = 0.447 + 0.316i$

$c_2 = 0.500 - 0.316i$

$c_3 = 0.387 + 0.448i$

(a) Verify that $|\alpha>$ is normalized.
(b) Write down the expression for $<\alpha|$.
(c) Write down the completeness relation.
(d) Find out the matrix representation of Hamiltonian operator, $|\alpha>$ and $<\alpha|$ in the given basis.

T6.2 The definition of Hermitian operator is $<\alpha|\hat{A}\beta> = <\hat{A}\alpha|\beta>$, where $|\alpha>$ and $|\beta>$ represent some arbitrary states.

(a) Show that sum of two Hermitian operators is Hermitian.
(b) Given $\hat{A}$ is Hermitian and c is any complex number, under what condition $c\hat{A}$ is Hermitian?
(c) Under what condition, the product of two Hermitian operators is Hermitian?

T6.3 Show that e^x and e^{-x} are eigenfunctions of the operator $\frac{d^2}{dx^2}$. Can we construct linear combinations of e^x and e^{-x} such that they are also eigenfunctions of the given operator?