A Novel Multipath Mitigation Technique for GNSS Signals in Urban Scenarios

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***Abstract*—In typical urban or suburban environments, multipath poses a serious threat to the accuracy of position determined by a global navigation satellite system (GNSS) receiver. In this paper, a novel multipath mitigation technique is proposed for general multipath scenarios encountered in the urban settings. In the proposed technique, a double differentiated correlation function based histogram output is used to estimate the code delay. It is shown that the proposed technique provides significant improvements over the current state of the art techniques like high resolution correlator/double delta correlator and narrow correlator. The results hold for general multipath models for the mobile-satellite channels. The performance guarantees include closed form expressions for the probability of correct delay estimate and simulation results for the average range error variation with respect to carrier to noise ratio (CNR), signal bandwidth and different channel settings.**

***Index Terms*—Multipath, Correlation, Code Delay, Double Differentiation, Probability, Histogram, Channel Coherence Time, Range Error.**

I. Introduction

In urban and suburban settings, presence of multiple reflectors and scatterers like high-rise buildings, towers, vehicles, trees cause the transmitted signal to reach the receiver through multiple paths. The average power profile of these multipath components including line of sight (LOS) component and other delayed replicas depend on the distribution of scatterers and reflectors surrounding the navigation receiver [1], [2], [3], [4], [5], [6]. Global Navigation Satellite System (GNSS) receiver while tracking the navigation signal can potentially get locked to any of the signal replicas coming from the transmitter, leading to a range error in the navigation solution [5], [6].

In GNSS receiver, range calculation requires an estimate of code phase delay of the received signal. The code phase delay is estimated by correlating the reference signal generated at the receiver with the incoming signal. For GNSS signals with rectangular pulse shape and binary phase shift keying (BPSK) modulation, the ideal correlation function is shown in Fig. 1. For code delay estimation, two parallel correlators with a relative spacing of one chip, known as early (E) and late (L) correlators, are used. As shown in discriminator function of Fig. 1, the code phase delay is estimated as the midpoint of E and L spacing when the difference of powers from E and L

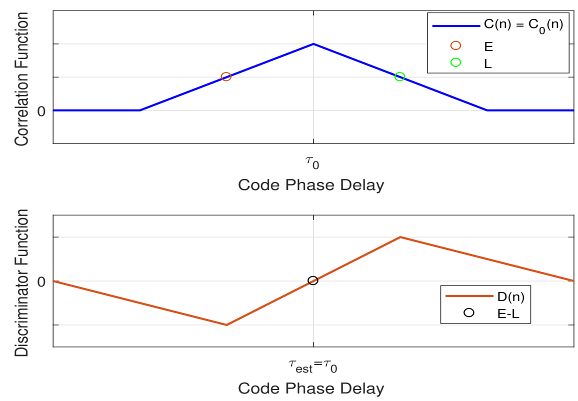

Figure 1. Correlator ($C(n)$) and discriminator ($D(n)$) functions with only LOS present: Resultant correlation function is due to LOS only and discriminator gives correct phase delay estimate.

When additional paths are received along with the LOS signal, correlation output is effectively the sum of correlations due to LOS signal and due to additional received signals. The impact of multipath on correlation output is that its symmetry gets disturbed. Due to disturbed symmetry, when E and L correlator outputs are equal, mid-point of E and L no longer corresponds to the correct phase delay and error is introduced in the computed code phase.

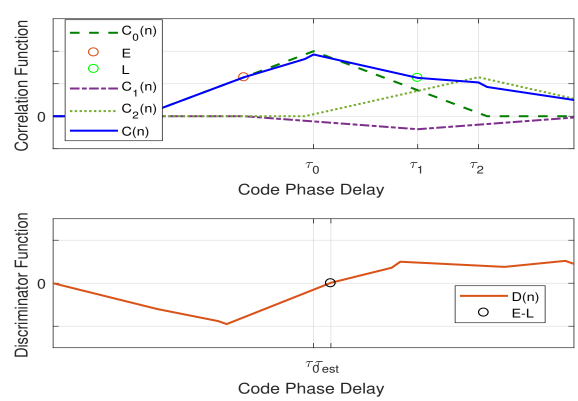

Figure 2. Correlator ($C(n)$) and discriminator (D(n)) functions with additional delayed replicas along with LOS component: resultant correlation function ($C(n)$) is sum of individual correlation functions ($C_0(n)$ due to LOS, $C_1(n), C_2(n)$ due to delayed replicas) due to multipath components and discriminator gives incorrect code phase delay ($\tau_{est} \neq \tau_0$).

Fig. 2 depicts the distorted correlation function through an example of a multipath signal consisting of LOS component

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and two delayed replicas. As can be seen, when E and L correlation values are equal, mid-point of E and L does not correspond to actual delay. Due to erroneous code phase, an error is introduced in the estimated range. In dense urban environments, there can be many number of additional delayed replicas along with LOS component, causing asymmetry in correlation function which eventually leads to error in range calculation.

The range error caused due to multipath propagation can be dealt with in two possible ways- multipath estimation [7], [8], [9],[10] and multipath mitigation [11], [12], [13]. As the multipath estimation techniques include optimization of the conditional probability function over a range of multipath delays, amplitudes and phases; the computational complexity is very high. The complexity increases as the required resolution and number of multipath components increase. The estimated channel parameters change after a short span of time (called channel coherence time) in typical urban settings [14]. Therefore, channel parameters need to be estimated again after each coherence time which further increases the computational complexity of estimation techniques. Therefore, multipath mitigation techniques are preferred over multipath estimation techniques due to high computational burden of multipath estimation techniques [12]. The multipath mitigation techniques include narrow correlator (NC), high resolution correlator (HRC) or double delta correlator, Early-Late (E-L) slope technique, E1-E2 tracker, strobe correlator etc. In narrow correlator, chip spacing between early (E) and late (L) correlators is reduced to 0.1 chip which is 1 chip in standard correlators [15]. In double delta correlator, two additional correlators, very early(VL) and very late(VL) are used, discriminator is a combination of two differences (E-L) and (VE-VL) [16]. E1-E2 tracker uses two early correlators, it is considered that multipath has no effect in this range, based on output of these correlators, slope of correlation is determined and code phase delay is predicted [17]. In E-L slope technique, also called Multipath Elimination Technology (MET) [18], two early E1 and E2, and two late L1 and L2 correlators are used to determine the slope on both sides of the peak, these slopes are used to predict code delay. In strobe correlators [19],[20],[21], different reference waveforms are used for minimizing error in code phase calculation. In slope difference techniques [22], [23], [24], correlation function is double differentiated and then its maxima is used for delay estimation. A co-operative multipath mitigation technique is recently proposed in [2] where inputs from different sensors and other nodes are combined in post-processing mode in order to mitigate the multipath effects.

Multipath mitigation techniques mentioned above, reduce error in code delay calculation, but there is always some residual error in code phase. Moreover, most of the techniques are developed using a special case of multipath propagation where only one delayed replica is considered along with LOS component and that too with an assumption that LOS has higher amplitude as compared to the amplitude of the delayed replica. The channel models in [4], [3], [25], [26], [27] suggest that in most pertinent urban settings of multipath propagation, there can be many number of delayed replicas along with LOS and LOS may not always have the highest power among all the paths. Therefore, for urban settings, there is a need to develop novel solutions that can mitigate multipath effects for general multipath propagation scenarios.

In this paper, a novel multipath mitigation technique is presented that works well in general multipath scenarios of [4], [3], [25], [26], [27]. In the proposed technique, a double differentiated correlation function based histogram output is used to estimate the code delay. The statistical independence of channel parameters (which change after every coherence time) is exploited in order to improve the probability of correct estimate. It is shown that the proposed technique provides significant improvements over the current state of the art techniques like high resolution correlator/double delta correlator and narrow correlator. The performance improvements are demonstrated using closed form expressions for the probability of correct code phase estimate and also through simulation results for different operational conditions.

II. System Model

Received digitized multipath signal, from a particular satellite, can be written as:

$$y(n) = A_0 x(n-\tau_0) + \sum_{i=1}^{L-1} A_i x(n-\tau_i) + w(n) \quad (1)$$

where, L is total number of paths through which signal is arriving at receiver. $w(n)$ is the noise added to the signal during reception assumed to be additive white Gaussian (AWG). A_0 and τ_0 are respectively the complex channel coefficient and arrival delay corresponding to LOS path. A_i and τ_i are respectively the complex channel coefficient and delay corresponding to i^{th} delayed version of LOS signal. $x(n)$ is discrete version of the binary phase shift keying (BPSK) modulated signal transmitted from the satellite.

The correlation function ($C(n)$) calculated in the baseband processing part of the GNSS receiver corresponding to the received signal in (1) is given by:

$$C(n) = C_0(n) + \sum_{i=1}^{L-1} C_i(n) + v(n) \quad (2)$$

where $C_0(n)$ is correlation function due to LOS signal, $C_i(n)$ is correlation function due to i^{th} delayed version of LOS signal, v is the contribution of AWG noise. An ideal correlation function (i.e. $C_0(n)$) corresponding to LOS is shown in Fig. 1 and can be written as:

$$C_0(n) =$$

$$\frac{A_0}{N_c} r(n-(\tau_0-N_c)) - \frac{2A_0}{N_c} r(n-\tau_0) + \frac{A_0}{N_c} r(n-(\tau_0+N_c)) \quad (3)$$

where $r(n)$ is discrete unit ramp function ($r(n-\tau) \triangleq n-\tau;$ if $n \geq \tau,$ and $0;$ otherwise), N_c is number of samples in one chip of PRN code, τ_0 is the delay corresponding to arrival of LOS signal and A_0 is the LOS channel coefficient. Similarly, $C_i(n)$ can be written as:

$$C_i(n) =$$

$$\frac{A_i}{N_c}r(n-(\tau_i-N_c))-\frac{2A_i}{N_c}r(n-\tau_i)+\frac{A_i}{N_c}r(n-(\tau_i+N_c)); \quad (4)$$
$$\text{for } 1 \leq i \leq (L-1)$$

where τ_i and A_i are respectively the delay and channel coefficient corresponding to the i^{th} delayed version of LOS signal.

III. Theory and Principle of the Proposed Technique

The proposed technique is aimed at mitigating the effect of signal multipath components on the estimated code-delay. The paper presents a novel technique for multipath mitigation and derives the probability that the estimated code-delay corresponds to LOS path even in the urban scenarios with $L(\geq 2)$ multipath components. The proposed technique is based on double differentiation of the correlation function in which the estimated delay corresponds to the multipath component having the largest amplitude among all the components. In the fading channels, for each instantaneous realization of the channel, any of the multipath components can have the highest amplitude. Due to which the delay estimate of individual instantaneous channel realization can correspond to any of the signal paths. To improve the probability that the delay estimate corresponds to LOS, we utilize the channel coherence properties and propose a novel technique based on histogram that ensures that the delay estimate corresponds to LOS with significantly high probability. We show that the proposed method outperforms the state of the art methods NC and HRC for wide range of system parameters. The results are quantified using realistic and measurement based channel models [28], [29], [30], [31]. Subsection III.A describes the proposed algorithm. In subsection III.B, code delay estimation from double differentiated correlation function is explained and the theorem, that index of the maxima of double differentiated correlation function corresponds to the delay of the component with largest amplitude, is proposed and proved (proof in Appendix A). In subsection III.C, the probability of correctness for the delay estimation through double differentiation is calculated using the statistical channel model. In III.D, the probability of correctness after applying histogram is calculated as a function of number of delay measurements taken for the histogram and improvement in probability of correctness is shown. In III.E, the upper bound, on the number of measurements taken for histogram for different systems and scenarios, is found out.

A. *Proposed Algorithm*

The stepwise description of the proposed algorithm, which we call as "Double Differentiated Histogram (DDH)" multipath mitigation algorithm, is shown in Fig. 3, and explained below:

Step.I Digitized Baseband Signal is correlated with reference code for computing the correlation output $C(n)$.

Step.II Double differentiate the correlation output (as given in Appendix A).

$$Q(n) = C(n) - 2C(n-1) + C(n-2). \quad (5)$$

Step.III Calculate Delay based on the index of maximum magnitude of double differentiated correlation output.

$$\hat{\tau} = \underset{n}{\operatorname{argmax}} |Q(n)|. \quad (6)$$

Step.IV Take M number of delay measurements between two consecutive updates given to navigation processor. The l^{th} of these M measurements is denoted by $\hat{\tau}^l$. The array of these M measurements is shown below.

$$\{\hat{\tau}^1, \hat{\tau}^2, ...\hat{\tau}^M\}.$$

Step.V Prepare histogram based on these M measurements. Histogram values are given by:

$$H(\tau_i) = m_i/M \quad (7)$$

where m_i is the number of times $\hat{\tau} = \tau_i$.

Step.VI Choose delay corresponding to maximum magnitude in histogram and update navigation processor with this value.

$$\tau_{est} = \underset{i}{\operatorname{argmax}} H(\tau_i). \quad (8)$$

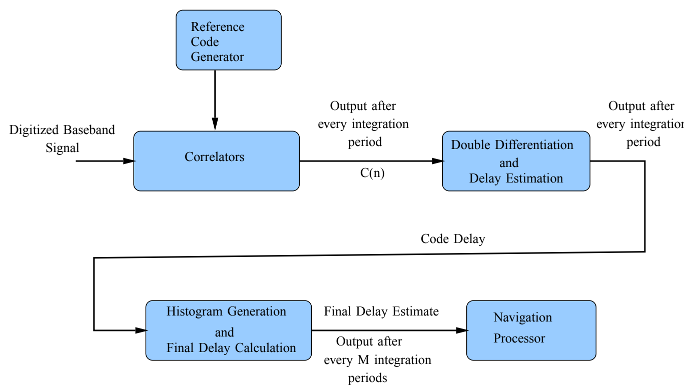

Figure 3. Schematic description of the proposed algorithm.

In **Step.I**, the complete correlation function is obtained using the multi-correlator approach [8], [10], [32]. The necessary resolution for the multiple correlators will become clear in the next section where the simulation results are discussed. The range taken for the correlators is one chip on either side of the current estimate of the LOS code-phase delay to ensure that the LOS path is not missed. In **Step.II** of the algorithm, the double differentiation of the correlation function is carried out. As the correlation function $C(n)$ also contains an AWGN term $v(n)$, double differentiating it will also results in double differentiation of the AWGN term (Appendix A). From the theory of random variables [33], it is known that the operation of double differentiation will make the noise power four times the original power in case of AWGN. Therefore, effective CNR will be reduced. The effect can be seen in the simulation results presented in the subsequent sections.

The theory behind estimating code delay from double differentiated correlator output and usefulness of histogram are explained in subsequent subsections.

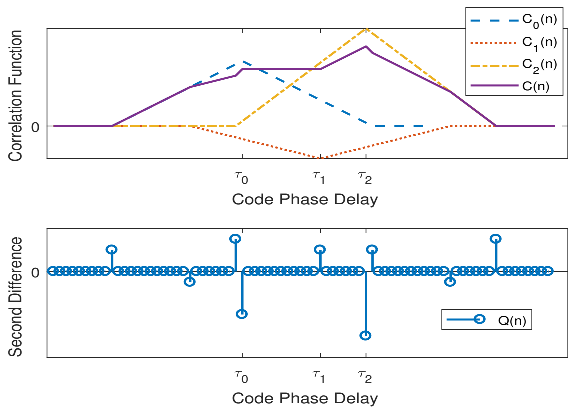

Figure 4. Correlator output with multipath and its second difference.

B. Code Delay Estimation

The proposed scheme includes computing second derivative of correlator output, which is equivalent to computing second difference of discrete correlator output. In principle (the ideal conditions when no noise is present), code delay estimation provides the delay corresponding to the path which is having maximum magnitude among all the paths as stated in the following theorem:

Theorem 1. *In ideal noise-free conditions, the maximum amplitude of the double differentiated correlator output occurs at the index which corresponds to the delay of the path with maximum magnitude i.e.*
If i^{th} path is having maximum magnitude among all the paths (i.e. $|A_i| \geq Max(|A_0|, |A_1|, ..., |A_{L-1}|)$), then:

$$\underset{n}{\operatorname{argmax}} |Q(n)| = \tau_i \tag{9}$$

where $Q(n)$ is the second difference of the resultant correlation function ($C(n)$) (shown in Fig. 4). Proof for theorem is given in Appendix A. □

C. Probability of Correct Estimate by Double Differentiated Correlator Output

For code delay estimate to be correct, amplitude of LOS signal should have maximum magnitude. But this may not always be the case as it depends purely on the channel. Many channel models are proposed in literature. The general approach to model a channel is recording data through large number of measurements and then making a statistical fit to these data. In the standard multipath channel model by [25], [28], [26]; channel coefficients are Rayleigh distributed and given as:

$$A_i \sim CN(0, \sigma_i^2); \tag{10}$$
$$\text{for } 1 \leq i \leq (L-1).$$

In satellite navigation channels, LOS channel coefficient (A_0) is Rician distributed while other A_i's are Rayleigh distributed [29], [34], [27]. A_0 is given as [14], [4]:

$$A_0 \sim \sqrt{\frac{k}{k+1}}\sigma_0 e^{j\theta} + \sqrt{\frac{1}{k+1}}CN(0, \sigma_0^2) \tag{11}$$

where, k is Rice factor which is the ratio of energy present in LOS path to the energy present in diffused paths. Satellite multipath channel models given in [35], [29], [28], [25] confirm that the power decay profile of multipath channel is exponential. Therefore, variance of different delayed paths can be given as:

$$\sigma_i^2 = \sigma_0^2 e^{-iT_s/T_{rms}}; \tag{12}$$
$$\text{for } 1 \leq i \leq (L-1).$$

and,

$$\sigma_0^2 = 1 - e^{-T_s/T_{rms}} \tag{13}$$

where T_s is sampling interval which depends on sampling frequency of receiver, T_{rms} is delay spread measure of multipath channel which depends on satellite elevation angle, user speed, and surrounding environment, its typical values can be found in [29], [35], [36], [37]. At each sample delay, starting from delay corresponding to LOS, there exists a signal path with an amplitude which is Rayleigh distributed random variable having variance as specified above.

Using the channel model described above, the probability (P_0) of LOS path having largest amplitude among all the paths can be given as:

$$P_0 = P[(|A_0| > |A_1|)\&(|A_0| > |A_2|)\&...\&(|A_0| > |A_{L-1}|)].$$

As amplitudes of these paths are random variables independent of each other, P_0 can be written in product form as:

$$P_0 = P(|A_0| > |A_1|).P(|A_0| > |A_2|)....P(|A_0| > |A_{L-1}|). \tag{14}$$

As $|A_i|$ is Rayleigh distributed random variable, its probability distribution function (PDF) is given by:

$$f_{|A_i|}(x) = \tfrac{2x}{\sigma_i^2} e^{-x^2/\sigma_i^2};\ x \geq 0.$$
$$\text{for } 1 \leq i \leq (L-1).$$

The PDF of Rician distributed A_0 is given by:

$$f_{|A_0|}(y) = \tfrac{2ky}{\sigma_0^2} e^{(-k-y^2k/\sigma_0^2)} I_0(\tfrac{2ky}{\sigma_0^2});\ y \geq 0$$

where, I_0 is the modified Bessel function zero order and first kind. Let's take p_i as the probability of $|A_i|$ being less than y, which is a particular value taken by random variable $|A_0|$. p_i can be calculated as:

$$p_i = \int_0^y f_{|A_i|}(x)dx = 1 - e^{-y^2/\sigma_i^2}; \tag{15}$$
$$\text{for } 1 \leq i \leq (L-1).$$

And P_0, from equation (14) is evaluated as:

$$P_0 = \int_0^\infty (\prod_1^{L-1} p_i) f_{|A_0|}(y)dy. \tag{16}$$

Through similar procedure, probability P_i of the i^{th} path being largest in magnitude can be calculated.
Probabilities, P_0, P_1, P_2 etc are computed taking $F_s = 20.46MHz$, which gives $T_s = 48.876ns$; $k = 3$ and $T_{rms} = 80ns$ which correspond to a typical urban environment [29], are $P_0 = 0.668$, $P_1 = 0.2286$, $P_2 = 0.0787$, $P_3 = 0.0195$, $P_4 = 0.0045$ and rest of the probabilities are almost zero.

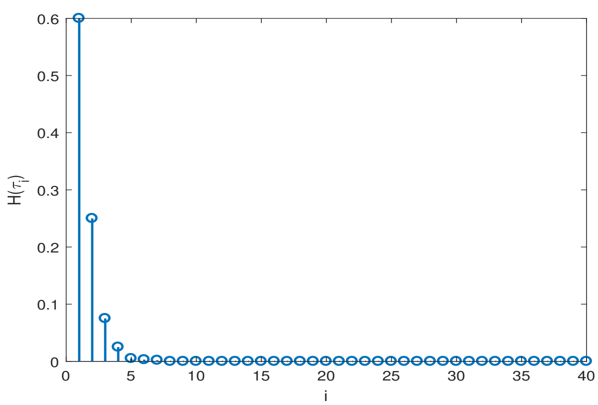

Figure 5. Histogram of code delay measurements given by maxima of double differentiated correlation function.

D. Improvement in Probability of Correct Estimate by using Histogram

We want to increase the probability of correct solution beyond P_0 to a particular desired value. For doing so, the fact utilized is that code delay solution needs to be fed to navigation processor (the block where final location calculation is done), only a few number of times in a second (lets say N times) or sometimes once in several seconds (where N can be less than 1). Therefore, code delay updates are required every $(1/N)$ seconds. During these $(1/N)$ seconds, correlator can be used to estimate code delay many number of times, lets say M number of times. A histogram is then prepared using these M observations. Based on the histogram the final delay value is fed to navigation processor. Histogram values are calculated as:

$$H(\tau_i) = m_i/M;$$

$$\text{for } 0 \leq i \leq (L-1).$$

where m_i is the number of times τ_i is the code delay prediction by double differentiator block. Such a histogram is shown in Fig. 5. The delay corresponding to maxima of histogram is taken as final delay solution. The probability of τ_0 (delay corresponding to LOS path) being the final code delay solution is:

$$P(\tau_0) = p(m_0 > m_i)$$

where $i = 1, 2...., L-1$ and

$$m_0 + m_1 + ... + m_{L-1} = M.$$

$P(\tau_0)$ is the improved probability of correct estimate can be calculated using the theorem given below.

Theorem 2. *If an event has L different outcomes with probabilities $P_0, P_1, ..., P_{L-1}$ respectively and is repeated M number of times, then the probability (p_p) of occurrence of a particular set of values $[m_0, m_1, ...m_{L-1}]$ is given by:*

$$p_p = \frac{M!}{m_0!m_1!...m_{L-1}} P_0^{m_0} P_1^{m_1} ... P_{L-1}^{m_{L-1}} \tag{17}$$

where, m_i is the number of times the outcome with probability P_i occurs, satisfying the condition $m_0+m_1+...+m_{L-1} = M$. The result stated in (17) is a standard result for generalization of Bernoulli trials derived in [33]. □

P_0, as stated before is the probability of LOS path having largest magnitude, P_1 is the probability that multipath component arriving one sample after the LOS is having largest magnitude and so on. P_0 is derived and calculated in previous section and P_1, P_2 etc can be derived and calculated similarly. For finding out $P(\tau_0)$, all p_p's corresponding to $m_0 \geq max(m_1, m_2, ...m_{L-1})$ are added. Therefore, $P(\tau_0)$ can be written as:

$$P(\tau_0) = \sum p_p[m_0 \geq max(m_1, m_2, ...m_{L-1})]. \tag{18}$$

For $k = 3$ as seen in previous subsection, $P_0 = 0.668$. Now using $M = 10$, we found $P(\tau_0) = 0.9531$; for $M = 20$, $P(\tau_0) = 0.9767$; while for $M = 5$, $P(\tau_0) = 0.8969$. Thus, the increase in the probability of correct solution by using histogram method is evident.

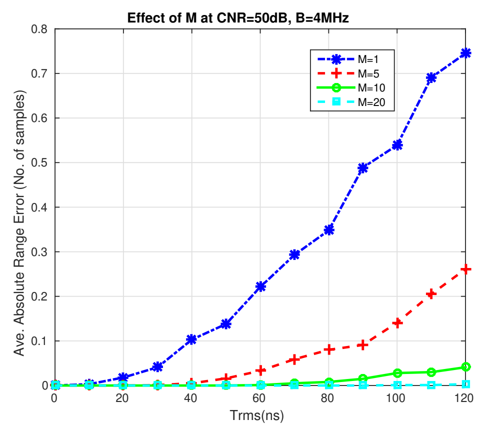

Figure 6. Range error v/s T_{rms} for proposed algorithm at different values of M.

We also show this through simulations by using the channel model given in [28], [29] where the channel behavior is controlled by the parameter T_{rms}. Typical values of T_{rms} for different types of environments like urban, suburban, rural, open etc can be found using the maximum delay values given in [29] which comes out to be $70-100ns$ for typical urban environments. For obtaining a feasible M, the filter bandwidth is fixed at $9MHz$ and for a fixed Carrier to noise ratio (CNR), range error (error which is introduced due to multipath) is plotted with T_{rms} for different values of M in Fig. 6. Other parameters (filter bandwidth and carrier to noise ratio) used for plotting are described in detail in the next section. The results in the plot agree with the fact that the probability of correct solution increases with increasing values of M.

E. Selection of histogram parameter-M

Two conditions should be ensured while making use of M code delay measurements from double differentiated correlator output for histogram preparation:

a. The line of sight (LOS) code-phase delay should remain constant during the M measurements used for histogram.
b. The M measurements should be statistically independent.

The analysis for possible values of M and statistical independence of code delay measurements is given below:
As shown in Fig. 3, code delay estimation is fed to the 'Histogram block' after every integration period (integration period is equal to the code period in a standard receiver i.e. $1/(MN) = 1$ Code Period) and the output of the histogram block is updated after every M integration periods. For the LOS code-phase delay to remain constant during these M integration periods, change in the distance between satellite and receiver during M integration periods should be less than the distance which will cause the signal delay to change by one sample. Therefore, following condition must be satisfied:

$$v_{s2r} M T_{code} < c T_s \tag{19}$$

where, v_{s2r} is the relative velocity between satellite and receiver, T_{code} is the code period, c is the speed of signal from satellite to receiver and T_s is the sampling period.
If the above condition is satisfied, the LOS code-phase delay will stay constant during the M measurements taken from "Double Differentiation and Delay Estimation". Due to the condition given above, M will be bounded by:

$$M < \frac{c}{v_{s2r} T_{code} F_s} \tag{20}$$

where, $F_s (= \frac{1}{T_s})$ is the sampling frequency.

The relative velocity for a stationary user ($v_{S_{s2r}}$) depends on the elevation angle of the satellite and can be found out using the formulation given in [5], as:

$$v_{S_{s2r}} = (v_{sat}) \times \tag{21}$$

$$sin\Big[tan^{-1}\Big(tan\theta + \frac{R_{earth} sec^2\theta}{\sqrt{R_s^2 sec^2\theta - R_{earth}^2} - R_{earth} tan\theta}\Big) - \theta\Big] \tag{22}$$

where, θ is the elevation angle of the satellite, v_{sat} is the speed of satellite considering the earth to be fixed (ECEF coordinates), R_{earth} is radius of the earth, and R_s is satellite orbital radius.

For a user in motion, the worst case can be assumed when the user velocity is in the same direction as that of the vector joining user and satellite, and gets added to the previously calculated relative speed for stationary user i.e. $v_{S_{s2r}}$. Assuming a typical urban user moving with a speed of $50 Kmph$, the worst case relative velocity can be given as:

$$v_{s2r\ \text{typical}} = v_{S_{s2r}} + 50 \times \frac{1000}{3600} \tag{23}$$

where the last term includes the factor for conversion of user speed from $Kmph$ to m/s and $v_{S_{s2r}}$ is relative speed for stationary user. Using the v_{sat} and R_s for different constellations (GPS, GLONASS, Galileo, BeiDou, NavIC etc.) and the formulations described above, the worst case values of M for a typical urban user at different elevation angles are calculated and given in Table I (other parameters used are $T_{code} = 1ms$, $F_s = 20.46 MHz$, $R_{earth} = 6400 Km$).

Now, assume a user moving with a high speed of $360 Kmph$ (typically the case for high speed railway networks).The worst case relative velocity in this scenario of a high speed user becomes:

Table I
WORST CASE VALUES OF M FOR DIFFERENT CONSTELLATIONS.

For a typical urban user moving at $50 Kmph$

θ	GPS	GLONASS	Galileo	BeiDou	NavIC
0°	18.69	15.97	22.13	20.14	34.06
10°	18.98	16.21	22.47	20.44	34.57
20°	19.87	16.98	23.52	21.40	36.17
30°	21.52	18.39	25.47	23.18	39.13

For a high-speed user moving at $360 Kmph$

θ	GPS	GLONASS	Galileo	BeiDou	NavIC
0°	16.84	14.60	19.58	18.01	28.36
10°	17.07	14.80	19.85	18.25	28.71
20°	17.79	15.44	20.66	19.01	29.81
30°	19.11	16.60	22.15	20.4	31.79

$$v_{s2r\ \text{high speed}} = v_{S_{s2r}} + 100 \tag{24}$$

where $100 m/s$ corresponds to the user speed of $360 Kmph$ and $v_{S_{s2r}}$ is relative speed for stationary user. The worst case values of M for a high speed user at different elevation angles are calculated and given in Table I. It can be seen from Equations (20), (23), (24) and Table I that an increase in the relative velocity $v_{S_{s2r}}$ decreases the value of the parameter M.

As can be seen from the Table I, values of $M = 10$ and $M = 20$ are reasonable for land-based high speed users (vehicles with speeds as high as $360 Kmph$). We have used $M = 10$ for the simulations (simulation results are presented in the forthcoming sections).

Now, as it is clear that M measurements are taken at different times with in $1/N$ seconds, we analyze the statistical independence of these M different measurements. The wireless channel is assumed to be almost constant during a short period called channel coherence time (T_{coh}) [14]. T_{coh} can be written in terms of v_{s2r} using the formulation given in [14] as:

$$T_{coh} = \frac{c}{4 f_c v_{s2r}} \tag{25}$$

where c is the speed of signal, f_c is the frequency of transmission, and v_{s2r} is relative velocity between satellite and receiver. For an example, GPS L1 signals can be considered, where $f_c = 1575.42 MHz$ and $T_{code} = 1ms$. For measurements to be statistically independent, following condition should be satisfied:

$$T_{coh} < 1ms \tag{26}$$

that puts a condition on elevation angle (θ) given below:

$$\theta < 80^\circ \tag{27}$$

which means that the measurements are statistically independent at elevation angles lower than 80° (T_{coh} ranges from few μs to few hundred μs). For higher elevation angles, the value of T_{coh} can be higher than $1ms$ ($2.5ms$ for typical urban settings as given in [14]) but as LOS component is very dominant at such high elevation angles, therefore even if effective M decrease, the accuracy of solution is much superior to that at the lower elevation angles.

IV. Effect of Filtering and Noise

In our algorithm code delay is estimated through double differentiation of correlator output, for the estimation to work correctly, peaks in the correlator output need to be preserved. Due to low pass nature of channel and filter used in RF front end of receiver, peaks in the correlator output get flattened. Noise which we ignored in the previous section, can not be avoided during the reception and affects the peaks present in correlator output. It can cause a peak to go down or can create a false peak and it's effect is much severe when double differentiation operation is used. Due to these effects, error is introduced in delay estimation which produces range error in location solution determined by navigation processor.

For analyzing the effect of filtering and quantifying the performance of the proposed technique in the presence of noise, we used MATLAB implementation of algorithm and signals simulated with different noise levels and different filter bandwidths. Average of the absolute range error is plotted, with carrier to noise ratio (CNR) for different filter bandwidths, at T_{rms} of 80 ns which corresponds to a typical urban environment [29], and for a sampling frequency $F_s = 20.46MHz$, in the Fig. 7. Absolute range error in terms of number of samples is calculated as:

$$Absolute\ Range\ Error = |Estimated\ Code\ Delay - Actual\ Code\ Delay|.$$

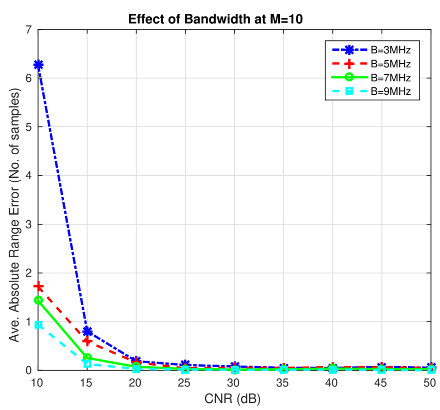

Figure 7. Range error v/s CNR for proposed algorithm at different bandwidths at $T_{rms} = 80ns$.

It can be observed from the Fig. 7 that range error increases with decreasing CNR i.e. with increasing noise power. The effect of bandwidth is also shown in the figure, range error is more for lower bandwidths as compared to higher bandwidths. At higher CNR, noise power is already small, but signal power entering in the system is more when a filter with higher bandwidth is used, thus range error is less for higher filter bandwidths. As can be seen from Fig. 7, for minimal error at moderate CNRs, a bandwidth of $9MHz$ is necessary which puts a limit on sampling frequency i.e. $F_s > 18MHz$. For simulations as stated above, we have taken $F_s = 20.46MHz$ which provides a resolution of 20 samples per chip. Therefore, the number of correlators used for obtaining the complete correlation function (one chip on either side) is 40.

V. Performance Comparison with existing state of the art schemes

Using GNSS receiver implemented in MATLAB according to our algorithm, BPSK signal structure and channel model given in previous sections; we compared the performance of our algorithm with existing popular schemes. Narrow correlator (NC) and double delta or high resolution correlator (HRC) are the two existing schemes that are widely used for multipath mitigation. Parameters taken are filter bandwidth $B = 9MHz$, number of measurements taken for histogram $M = 10$, channel rice factor of LOS signal $k = 3$ and sampling frequency of the system $F_s = 20.46MHz$. For these given conditions ranging error for narrow correlator, high resolution correlator and for purposed scheme are calculated and compared at different value of CNR. For narrow correlator (NC), discriminator output is $E - L$ and chip spacing between early and late correlators is 0.1 chips. The discriminator used for HRC is:

$$discriminator\ output = (E - L) + \frac{1}{2}\ (VE - VL).$$

For HRC, chip spacing between early and late is 0.1 chips and chip spacing between very early and very late is 1 chip.

For the parameter $M = 10$, the proposed technique provides output after every 10 code periods ($10ms$ in case of GPS C/A and NavIC). Therefore, an averaging of 10 delay measurements for NC and HRC is also taken for a fair comparison. Range Error after 10 code period averaging for NC, HRC and proposed algorithm is plotted in Fig. 8 and 9 at different CNRs, number of realizations taken for averaging is 1000. It is evident from these figures that the purposed technique outperforms the existing techniques at high CNRs ($50dB$), as well as at low CNRs ($20dB$).

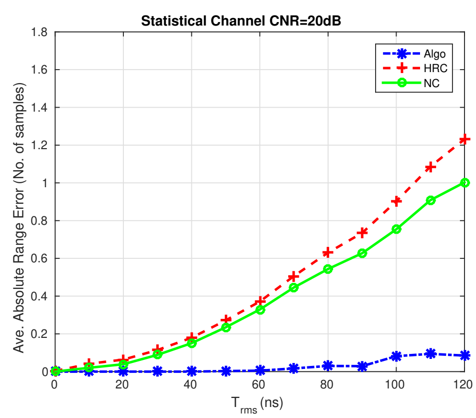

Figure 8. Range error v/s T_{rms} for NC, HRC and proposed algorithm at CNR = 20 dB, B=9MHz and M=10 for statistical model [28], [29].

VI. Performance Verification using Land Mobile Satellite Channel Model

For performance verification, we have used the land mobile satellite (LMS) channel model developed through field trials by DLR Germany and also standardized by International

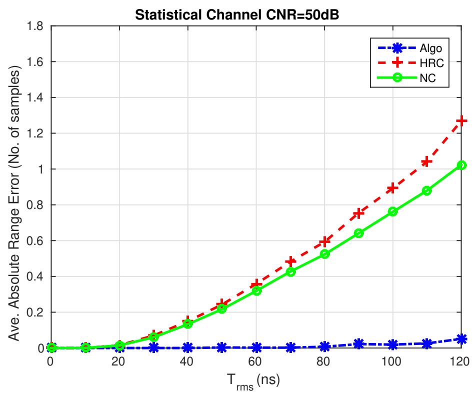

Figure 9. Range error v/s T_{rms} for NC, HRC and proposed algorithm at CNR = 50 dB, B=9MHz and M=10 for statistical model [28], [29].

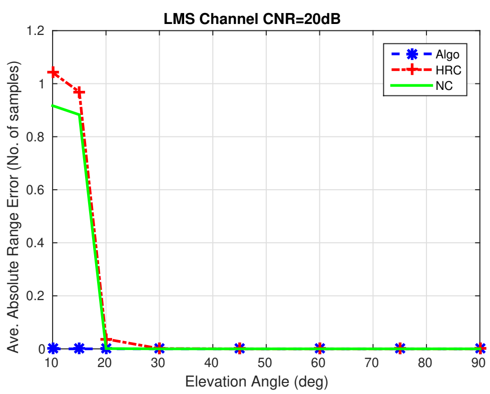

Figure 10. Range error v/s T_{rms} for NC, HRC and proposed algorithm at CNR = 20 dB, B=9MHz and M=10 for LMS model [35], [38], [31].

Telecom Union (ITU) [35], [38], [31]. The MATLAB implementation of the model was accessed from Institute of Communications and Navigation, DLR, Germany. The model is utilized using the technical notes from DLR [39]. More information about the model can be found on [30]. The model simulates the obstacles encountered by the signal (tree, house, poles etc.), takes in to account the elevation angle of the satellite and the ground user speed, and generates channel coefficients for LOS and different multipath components. At lower elevation angles the signal encounters more obstacles and more number of multipath components are produced as compared to the higher elevation angles. Similarly as higher T_{rms} generates more number of multipath components in statistical model, lower elevation angles in LMS model generate more number of multipath and also reduce the power on LOS component. The performance of the proposed algorithm is assessed using LMS model at different signal strengths (CNRs) and compared with NC and HRC. Fig. 10 and 11 show the performance comparison of NC, HRC and proposed algorithm at different at low and high CNRs respectively, the range error is plotted with elevation angle at a typical urban user speed of $50Kmph$.

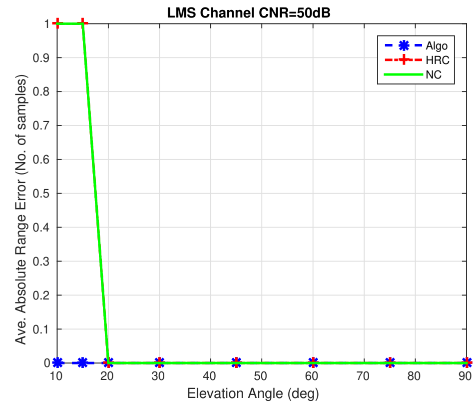

Figure 11. Range error v/s T_{rms} for NC, HRC and proposed algorithm at CNR = 50 dB, B=9MHz and M=10 for LMS model [35], [38], [31].

VII. Conclusion

It is shown that the proposed technique, based on histogram of code delay given by double differentiated correlator output, is able to provide accurate output of code delay in pertinent multipath scenarios with a very high probability although it involves the computation of full correlation function. As shown in the paper, the proposed technique provides improved accuracy as compared to the currently known best multipath mitigation techniques. The performance guarantees hold for the CNRs of as low as $20dB$, for the typical vehicle speeds ($50Kmph$) as well as for high speeds ($360Kmph$), and for different multipath scenarios (generated by changing the elevation angle). As the parameters taken above correspond to urban, sub-urban, metropolitan environments and for moving vehicles where channel coherence time is small, the performance is expected to hold in these type of scenarios. The proposed technique will be useful in future generation smart vehicles and high accuracy based location services.

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Appendix A

Proof of **Theorem 1**

First difference of correlator output can be written as:

$$P(n) = \sum_{i=0}^{L-1}(C_i(n) - C_i(n-1)) + v(n) - v(n-1)$$

where,

$$C_i(n) - C_i(n-1) =$$

$$\Big(\frac{A_i}{N_c}r(n-(n_i-N_c)) - \frac{2A_i}{N_c}r(n-n_i) + \frac{A_i}{N_c}r(n-(n_i+N_c))\Big)$$

$$-\Big(\frac{A_i}{N_c}r(n-1-(n_i-N_c)) - \frac{2A_i}{N_c}r(n-1-n_i)+$$

$$\frac{A_i}{N_c} r(n-1-(n_i+N_c))\Big).$$

As we know that,

$$u(n) = r(n) - r(n-1)$$

where $u(n)$ is discrete unit step function. Therefore,

$$C_i(n) - C_i(n-1) =$$

$$\frac{A_i}{N_c} u(n-(n_i-N_c)) - \frac{2A_i}{N_c} u(n-n_i) + \frac{A_i}{N_c} u(n-(n_i+N_c)).$$

Now, $P(n)$ can be written as

$$P(n) =$$

$$\sum_{i=0}^{L-1} \frac{A_i}{N_c} u(n-(n_i-N_c)) - \frac{2A_i}{N_c} u(n-n_i) + \frac{A_i}{N_c} u(n-(n_i+N_c))$$

$$+v(n) - v(n-1).$$

Second difference of correlator output can be written as:

$$Q(n) = P(n) - P(n-1).$$

As we know that,

$$\delta(n) = u(n) - u(n-1)$$

where, $\delta(n)$ is discrete unit impulse function. Therefore,

$$Q(n) =$$

$$\sum_{i=0}^{L-1} \frac{A_i}{N_c} \delta(n-(n_i-N_c)) - \frac{2A_i}{N_c} \delta(n-n_i) + \frac{A_i}{N_c} \delta(n-(n_i+N_c))$$

$$+v''(n).$$

where, $v''(n)$ i.e. noise in double differentiated correlator output, can be written as (following the above double differentiation operation):

$$v''(n) \triangleq v(n) - 2v(n-1) + v(n-2).$$

It can be seen from this equation that LOS signal and each of its delayed versions contribute three delta (impulse) terms to the second difference of correlator output. Two of these three deltas which are N_c samples before and after the central delta, are having same magnitude. The magnitude of central delta is double of the magnitude of a side delta, this is shown in Fig. 4. From the figure and above equations it is clear that in noise-free conditions (i.e. $v(n=0)$ for all the values of n and therefore, $v''(n) = 0$), if k^{th} path is having maximum amplitude among all other paths i.e.

$$|A_k| > |A_i|; \text{ for } i = 0, 1, 2..., L-1 \text{ and } i \neq k.$$

then,

$$|2A_k/N_c| > |2A_i/N_c|.$$

Therefore, the maximum value of attained by the magnitude of second difference is $2A_k/N_c$ and this value occurs at the delay corresponding to arrival of the k^{th} path. This fact is also confirmed in [22], [23], [24].

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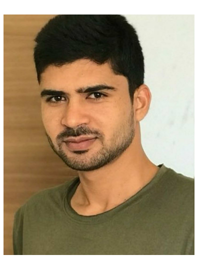

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