

Indian Institute of Technology Jodhpur, Year 2018-2019

Digital Logic and Design

(Course Code: EE222)

Lecture 20-21: Counters contd....

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Course related documents will be uploaded on
<http://home.iitj.ac.in/~sptiwari/DLD/>

Note: The information provided in the slides are taken from text books Digital Electronics (including Mano & Ciletti), and various other resources from internet, for **teaching/academic use only**

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Counters Overview

- **Counters are important components in computers**
 - The increment or decrement by one in response to input
- **Two main types of counters**
 - Ripple (asynchronous) counters
 - Synchronous counters
- **Ripple counters**
 - Flip flop output serves as a source for triggering other flip flops
- **Synchronous counters**
 - All flip flops triggered by a **clock** signal
- **Synchronous counters are more widely used in industry.**

Counters

- **Asynchronous counters:** the flip-flops do not change states at exactly the same time as they do not have a common clock pulse.
- Known as **ripple counters**, as the input clock pulse “ripples” through the counter – cumulative delay is a drawback.
- n flip-flops → a MOD (modulus) 2^n counter. (Note: A MOD- x counter cycles through x states.)
- Output of the last flip-flop (MSB) divides the input clock frequency by the MOD number of the counter, hence a counter is also a *frequency divider*.

Counters

- **Counter:** A register that goes through a prescribed series of states
- **Binary counter**
 - Counter that follows a binary sequence
 - N bit binary counter counts in binary from n to $2^n - 1$
- **Ripple counters** triggered by initial **Count** signal
- **Applications:**
 - Watches
 - Clocks
 - Alarms
 - Web browser refresh

Asynchronous Counters

- Each FF output drives the CLK input of the next FF.
- FFs do not change states in exact synchronism with the applied clock pulses.
- *There is delay between the responses of successive FFs.*
- *Ripple counter* due to the way the FFs respond one after another in a kind of rippling effect.

A ₃	A ₂	A ₁	A ₀
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
1	0	0	0
1	0	0	1

Binary Ripple Counters with T and D-FF

- Reset signal sets all outputs to 0
- Count signal toggles output of low-order flip flop
- Low-order flip flop provides trigger for adjacent flip flop
- Not all flops change value simultaneously
 - Lower-order flops change first
- Focus on D flip flop implementation

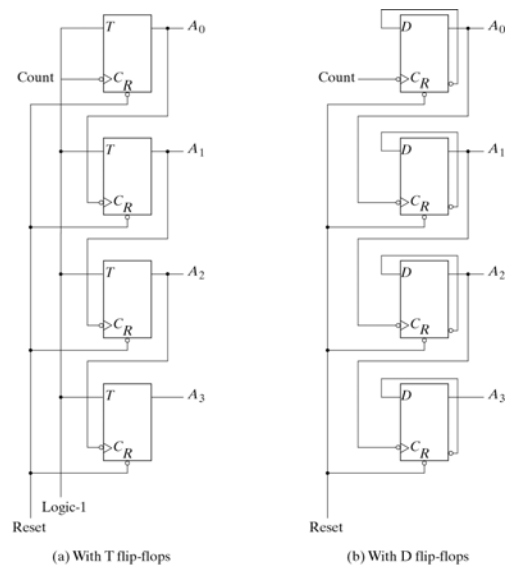
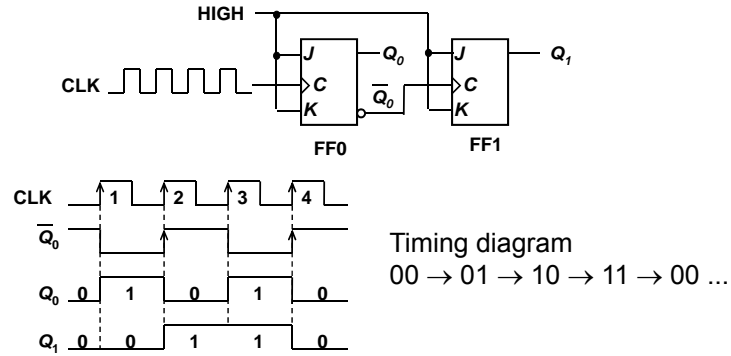


Fig. 6-8 4-Bit Binary Ripple Counter

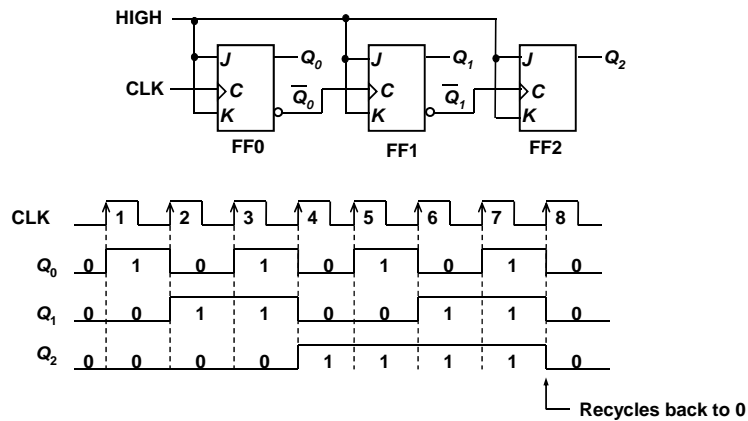
Asynchronous Counters

- Example: 2-bit ripple binary counter.
- Output of one flip-flop is connected to the clock input of the next more-significant flip-flop.



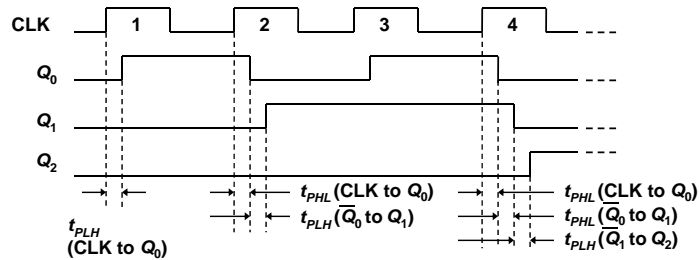
Asynchronous Counters

- Example: 3-bit ripple binary counter.



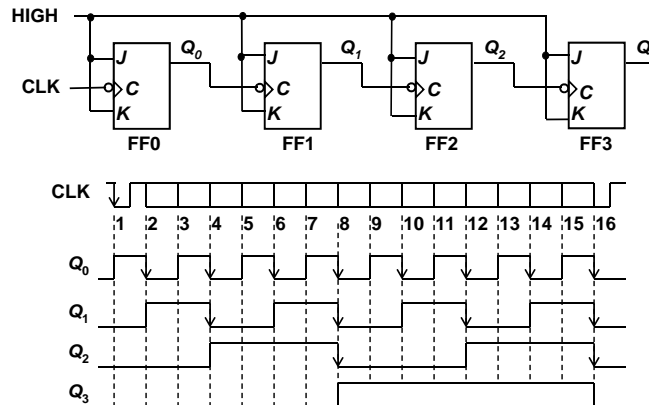
Asynchronous Counters

- Propagation delays in an asynchronous (ripple-clocked) binary counter.
- If the accumulated delay is greater than the clock pulse, some counter states may be misrepresented!



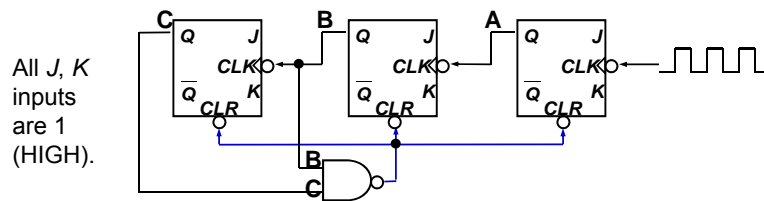
Asynchronous Counters

- Example: 4-bit ripple binary counter (negative-edge triggered).



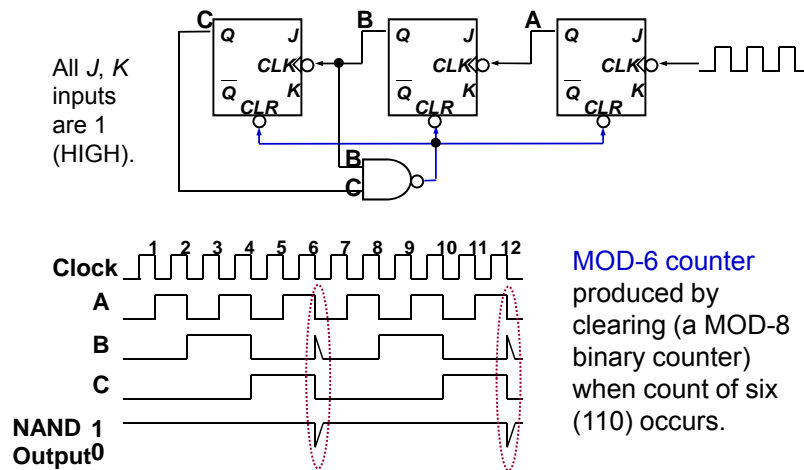
Asynchronous Counters with MOD no. $< 2^n$

- States may be skipped resulting in a **truncated sequence**.
- **Technique:** force counter to *recycle before going through all of the states* in the binary sequence.
- **Example:** Given the following circuit, determine the counting sequence (and hence the modulus no.)



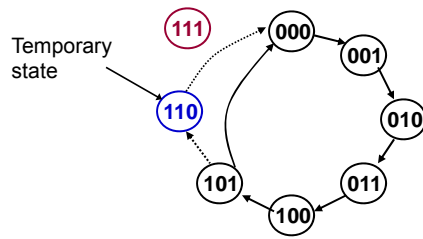
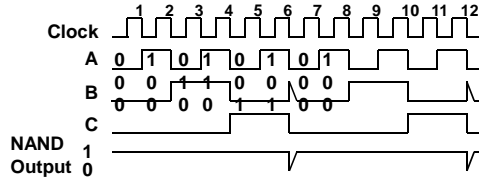
Asynchronous Counters with MOD no. $< 2^n$

- **Example (cont'd):**



Asynchronous Counters with MOD no. $< 2^n$

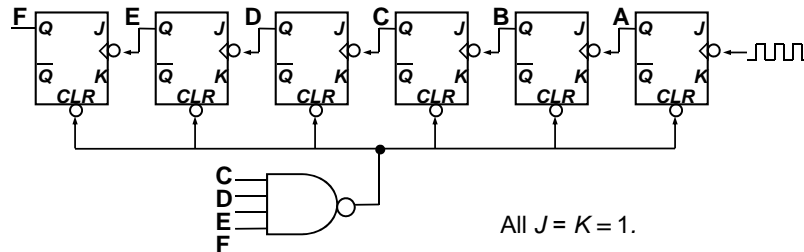
- **Example (cont'd):** Counting sequence of circuit (in CBA order).



Counter is a MOD-6 counter.

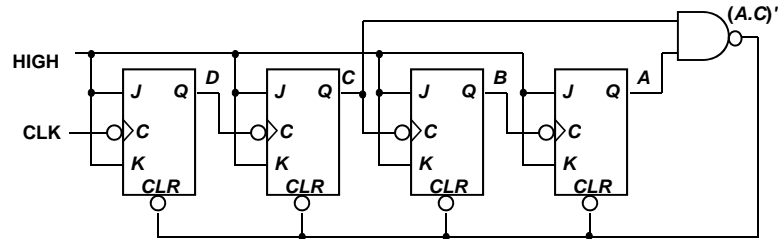
Asynchronous Counters with MOD no. $< 2^n$

- **Exercise:** How to construct an asynchronous MOD-5 counter? MOD-7 counter? MOD-12 counter?
- **Question:** The following is a MOD-? counter?



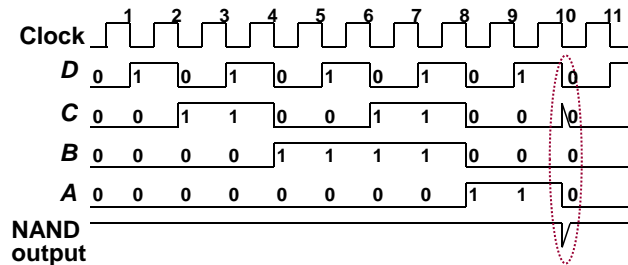
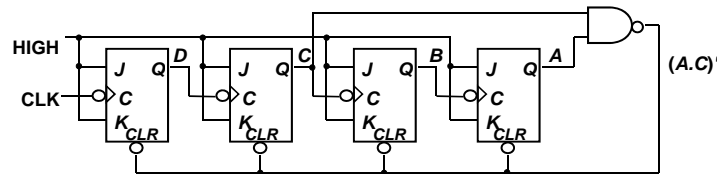
Asynchronous Counters with MOD no. $< 2^n$

- **Decade counters (or BCD counters)** are counters with 10 states (modulus-10) in their sequence. They are commonly used in daily life (e.g.: utility meters, odometers, etc.).
- Design an asynchronous decade counter.



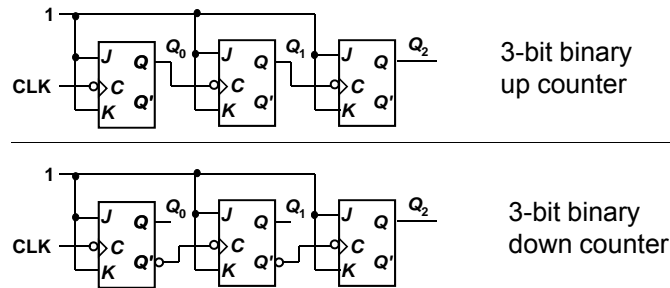
Asynchronous Counters with MOD no. $< 2^n$

- **Asynchronous decade/BCD counter (cont'd).**



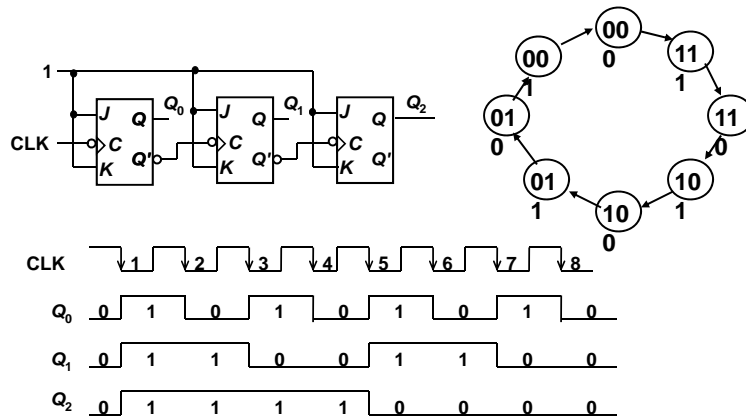
Asynchronous Down Counters

- So far we are dealing with *up counters*. **Down counters**, on the other hand, count downward from a maximum value to zero, and repeat.
- Example: A 3-bit binary (MOD- 2^3) down counter.



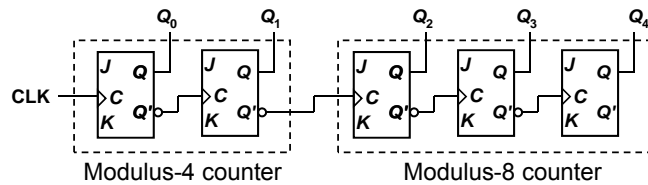
Asynchronous Down Counters

- Example: A 3-bit binary (MOD-8) down counter.



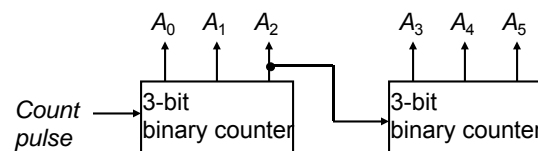
Cascading Asynchronous Counters

- Larger asynchronous (ripple) counter can be constructed by cascading smaller ripple counters.
- Connect last-stage output of one counter to the clock input of next counter so as to achieve higher-modulus operation.
- Example: A modulus-32 ripple counter constructed from a modulus-4 counter and a modulus-8 counter.



Cascading Asynchronous Counters

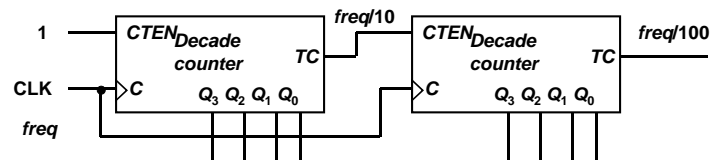
- Example: A 6-bit binary counter (counts from 0 to 63) constructed from two 3-bit counters.



A_5	A_4	A_3	A_2	A_1	A_0
0	0	0	0	0	0
0	0	0	0	0	1
0	0	0	:	:	:
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	0	0	1
:	:	:	:	:	:

Cascading Asynchronous Counters

- If counter is not a binary counter, requires additional output.
- Example: A modulus-100 counter using two decade counters.



TC = 1 when counter recycles to 0000

Synchronous counters

- **Synchronous(parallel) counters**
 - All of the FFs are triggered simultaneously by the clock input pulses.
 - All FFs change at same time
- **Remember**
 - If J=K=0, flop maintains value
 - If J=K=1, flop toggles
- **Most counters are synchronous in computer systems.**
- **Can also be made from D flops**
- **Value increments on positive edge**

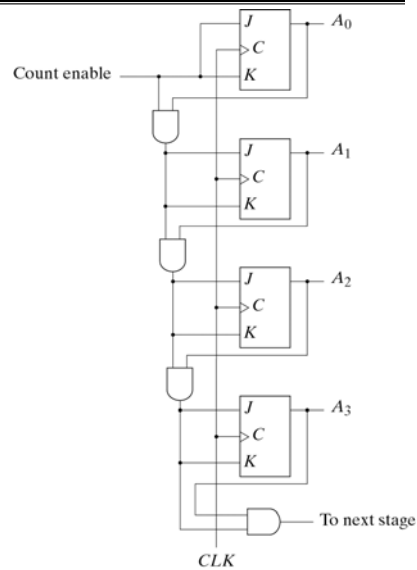
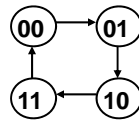


Fig. 6-12 4-Bit Synchronous Binary Counter

Synchronous (parallel) counters

- **Synchronous (parallel) counters:** the flip-flops are clocked at the same time by a common clock pulse.
- We can design these counters using the sequential logic design process (will be covered in coming Lectures).
- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).



Present state		Next state		Flip-flop inputs	
A_1	A_0	A_1^+	A_0^+	TA_1	TA_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

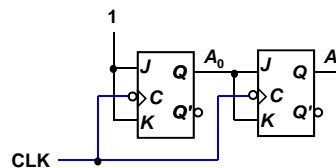
Synchronous (Parallel) Counters

- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).

Present state		Next state		Flip-flop inputs	
A_1	A_0	A_1^+	A_0^+	TA_1	TA_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

$$TA_1 = A_0$$

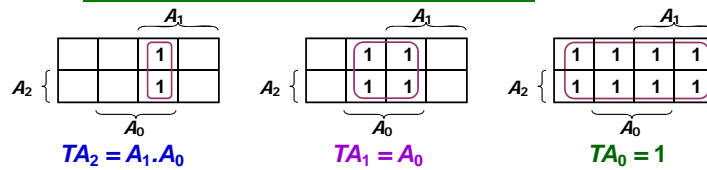
$$TA_0 = 1$$



Synchronous (Parallel) Counters

- Example: 3-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J, K inputs).

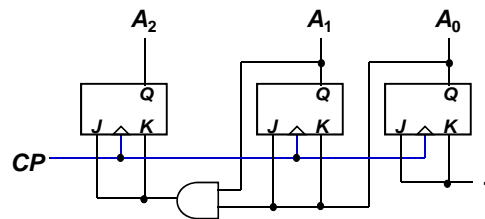
Present state			Next state			Flip-flop inputs		
A_2	A_1	A_0	A_2^+	A_1^+	A_0^+	TA_2	TA_1	TA_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1



Synchronous (Parallel) Counters

- Example: 3-bit synchronous binary counter (cont'd).

$TA_2 = A_1 \cdot A_0$ $TA_1 = A_0$ $TA_0 = 1$



Synchronous (Parallel) Counters

- Note that in a binary counter, the n^{th} bit (shown underlined) is always complemented whenever

$$\underline{0}11\dots11 \rightarrow \underline{1}00\dots00$$

$$\text{or } \underline{1}11\dots11 \rightarrow \underline{0}00\dots00$$

- Hence, X_n is complemented whenever

$$X_{n-1}X_{n-2} \dots X_1X_0 = 11\dots11.$$

- As a result, if T flip-flops are used, then

$$TX_n = X_{n-1} \cdot X_{n-2} \cdot \dots \cdot X_1 \cdot X_0$$

Synchronous (Parallel) Counters

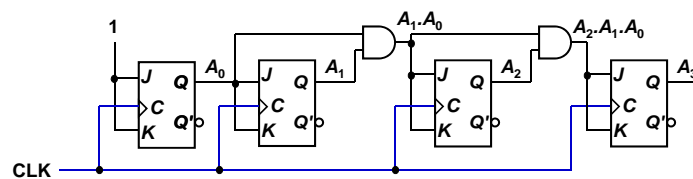
- Example: 4-bit synchronous binary counter.

$$TA_3 = A_2 \cdot A_1 \cdot A_0$$

$$TA_2 = A_1 \cdot A_0$$

$$TA_1 = A_0$$

$$TA_0 = 1$$



Synchronous (Parallel) Counters

Example: Synchronous decade/BCD counter.

Clock pulse	Q_3	Q_2	Q_1	Q_0
Initially	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10 (recycle)	0	0	0	0

$$T_0 = 1$$

$$T_1 = Q_3' \cdot Q_0$$

$$T_2 = Q_1 \cdot Q_0$$

$$T_3 = Q_2 \cdot Q_1 \cdot Q_0 + Q_3 \cdot Q_0$$

Synchronous (Parallel) Counters

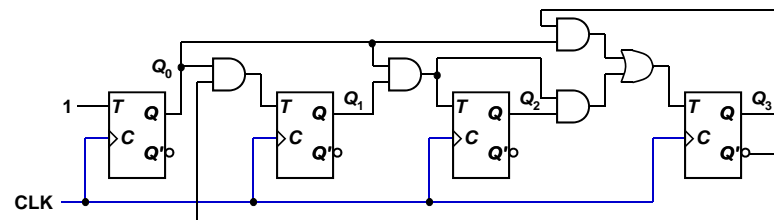
Example: Synchronous decade/BCD counter (cont'd).

$$T_0 = 1$$

$$T_1 = Q_3' \cdot Q_0$$

$$T_2 = Q_1 \cdot Q_0$$

$$T_3 = Q_2 \cdot Q_1 \cdot Q_0 + Q_3 \cdot Q_0$$



Synchronous UP/Down counters

- Up/Down Counter can either count up or down on each clock cycle
- Up counter counts from 0000 to 1111 and then changes back to 0000
- Down counter counts from 1111 to 0000 and then back to 1111
- Counter counts up or down each clock cycle
- Output changes occur on clock rising edge

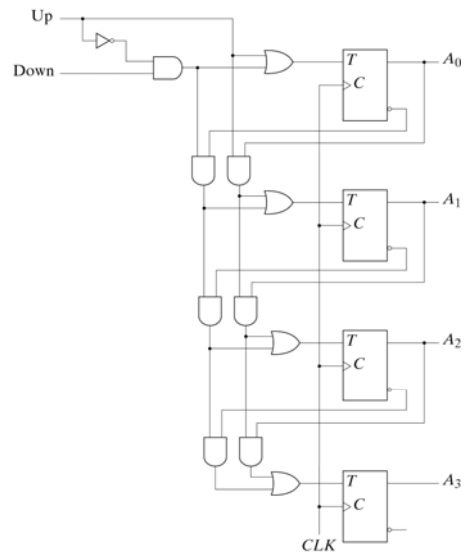


Fig. 6-13 4-Bit Up-Down Binary Counter

Up/Down Synchronous Counters

- **Up/down synchronous counter:** a *bidirectional* counter that is capable of counting either up or down.
- An input (control) line *Up/Down* (or simply *Up*) specifies the direction of counting.
 - ❖ *Up/Down* = 1 → Count upward
 - ❖ *Up/Down* = 0 → Count downward

Up/Down Synchronous Counters

- Example: A 3-bit up/down synchronous binary counter.

Clock pulse	Up	Q ₂	Q ₁	Q ₀	Down
0	↗	0	0	0	↘
1	↗	0	0	1	↘
2	↗	0	1	0	↘
3	↗	0	1	1	↘
4	↘	1	0	0	↗
5	↘	1	0	1	↗
6	↘	1	1	0	↗
7	↘	1	1	1	↗

$$TQ_0 = 1$$

$$TQ_1 = (Q_0 \cdot Up) + (Q_0' \cdot Up')$$

$$TQ_2 = (Q_0 \cdot Q_1 \cdot Up) + (Q_0' \cdot Q_1' \cdot Up')$$

Up counter

$$TQ_0 = 1$$

$$TQ_1 = Q_0$$

$$TQ_2 = Q_0 \cdot Q_1$$

Down counter

$$TQ_0 = 1$$

$$TQ_1 = Q_0'$$

$$TQ_2 = Q_0' \cdot Q_1'$$

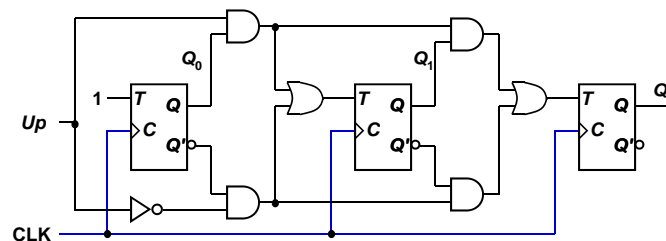
Up/Down Synchronous Counters

- Example: A 3-bit up/down synchronous binary counter (cont'd).

$$TQ_0 = 1$$

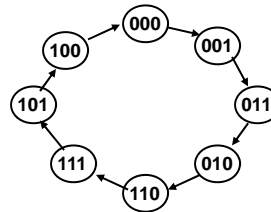
$$TQ_1 = (Q_0 \cdot Up) + (Q_0' \cdot Up')$$

$$TQ_2 = (Q_0 \cdot Q_1 \cdot Up) + (Q_0' \cdot Q_1' \cdot Up')$$



Designing

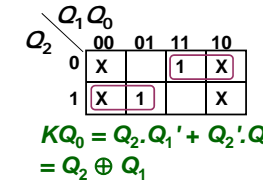
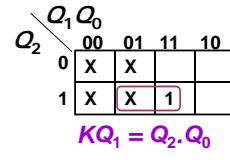
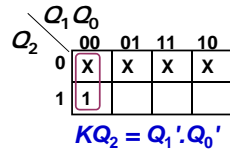
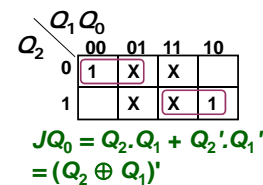
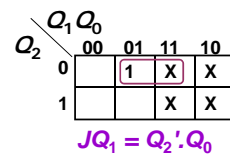
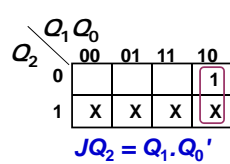
- As Covered Earlier
- Example: A 3-bit Gray code counter (using JK flip-flops).



Present state			Next state			Flip-flop inputs					
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	JQ_2	KQ_2	JQ_1	KQ_1	JQ_0	KQ_0
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	1	0	X	1	X	X	0
0	1	0	1	1	0	1	X	X	0	0	X
0	1	1	0	1	0	0	X	X	0	X	1
1	0	0	0	0	0	X	1	0	X	0	X
1	0	1	1	0	0	X	0	0	X	X	1
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	1	0	1	X	0	X	1	X	0

Up/Down Synchronous Counters

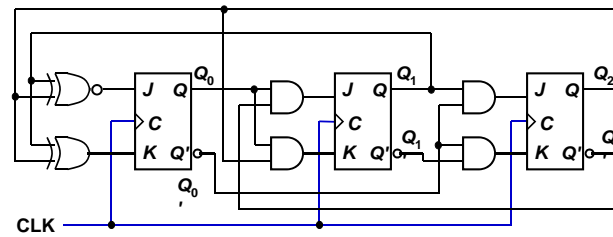
- 3-bit Gray code counter: flip-flop inputs.



Up/Down Synchronous Counters

3-bit Gray code counter: logic diagram.

$$\begin{array}{lll}
 JQ_2 = Q_1 \cdot Q_0' & JQ_1 = Q_2' \cdot Q_0 & JQ_0 = (Q_2 \oplus Q_1)' \\
 KQ_2 = Q_1' \cdot Q_0' & KQ_1 = Q_2 \cdot Q_0 & KQ_0 = Q_2 \oplus Q_1
 \end{array}$$



Counters with Parallel Load

- Counters with parallel load can have a preset value
- Load signal indicates that data ($I_3 \dots I_0$) should be loaded into the counter
- Clear resets counter to all zeros
- Carry output could be used for higher-order bits

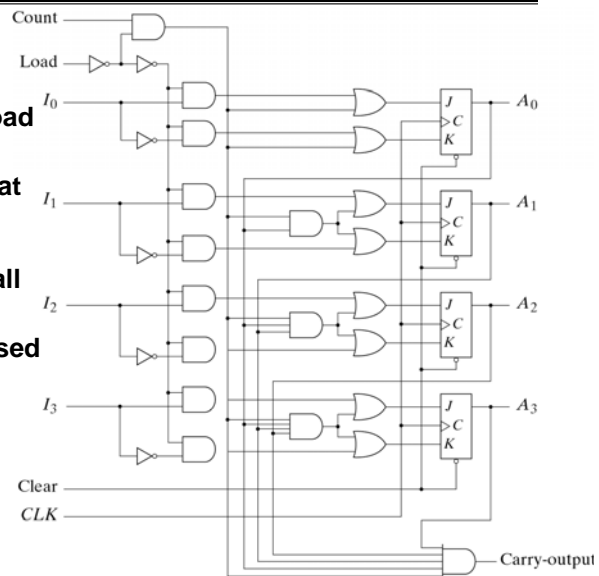


Fig. 6-14 4-Bit Binary Counter with Parallel Load

Counters with Parallel Load

Clear	Clk	Load	Count	Function
0	X	X	X	Clear to 0
1	↑	1	X	Load inputs
1	↑	0	1	Count
1	↑	0	0	No Change

Function Table

- If **Clear** is asserted (0), the counter is cleared
- If **Load** is asserted data inputs are loaded
- If **Count** asserted counter value is incremented

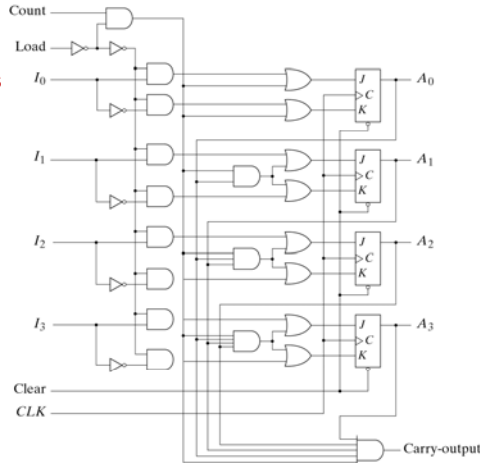
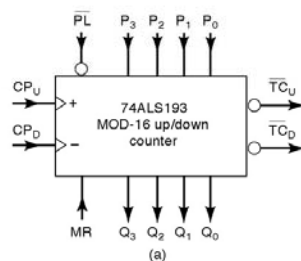


Fig. 6-14 4-Bit Binary Counter with Parallel Load

Binary Counter with Parallel Load and Preset

- Commercial version of binary counter



Mode Select				Mode
MR	PL	CP _U	CP _D	
H	X	X	X	Asynch. reset
L	L	X	X	Asynch. preset
L	H	H	H	No change
L	H	↑	H	Count up
L	H	H	↑	Count down

H = HIGH; L = LOW
X = Don't care; ↑ = PGT
(c)

Pin	Description
CP _U	Count-up clock input (active rising edge)
CP _D	Count-down clock input (active rising edge)
MR	Asynchronous master reset input (active HIGH)
PL	Asynchronous parallel load input (active LOW)
P ₀ -P ₃	Parallel data inputs
Q ₀ -Q ₃	Flip-flop outputs
TC _D	Terminal count-down (borrow) output (active LOW)
TC _U	Terminal count-up (carry) output (active) LOW

(b)

Summary

- **Binary counters can be ripple or synchronous**
- **Ripple counters use flip flop outputs as flop triggers**
 - Some delay before all flops settle on a final value
 - Do not require a clock signal
- **Synchronous counters are controlled by a clock**
 - All flip flops change at the same time
- **Up/Down counters can either increment or decrement a stored binary value**
 - Control signal determines if counter counts up or down
- **Counters with parallel load can be set to a known value before counting begins.**