Carrier Diffusion across the Junction

- Because of the difference in hole and electron concentrations on each side of the junction, carriers diffuse across the junction:

![Diagram of carrier diffusion across a junction](image)

**Notation:**
- $n_n$ = electron concentration on N-type side (cm$^{-3}$)
- $p_n$ = hole concentration on N-type side (cm$^{-3}$)
- $p_p$ = hole concentration on P-type side (cm$^{-3}$)
- $n_p$ = electron concentration on P-type side (cm$^{-3}$)
Depletion Region

- As conduction electrons and holes diffuse across the junction, they leave behind ionized dopants. Thus, a region that is depleted of mobile carriers is formed.
  - The charge density in the depletion region is not zero.
  - The carriers which diffuse across the junction recombine with majority carriers, i.e., they are annihilated.

Carrier Drift across the Junction

- Because charge density $\neq 0$ in the depletion region, an electric field exists, hence there is drift current.
PN Junction in Equilibrium

- In equilibrium, the drift and diffusion components of current are balanced; therefore the net current flowing across the junction is zero.

\[
J_{p,\text{drift}} = -J_{p,\text{diff}} \\
J_{n,\text{drift}} = -J_{n,\text{diff}} \\
J_{\text{tot}} = J_{p,\text{drift}} + J_{n,\text{drift}} + J_{p,\text{diff}} + J_{n,\text{diff}} = 0
\]

Built-in Potential, \( V_0 \)

- Because of the electric field in the depletion region, there exists a potential drop across the junction:

\[
q \mu_p E = qD_p \frac{dp}{dx} \Rightarrow p \mu_p \left( -\frac{dV}{dx} \right) = D_p \frac{dp}{dx}
\]

\[
\Rightarrow -\mu_p \int_{x_1}^{x_2} dV = D_p \int_{p_n}^{p_p} \frac{dp}{p}
\]

\[
\Rightarrow V(x_1) - V(x_2) = \frac{D_p}{\mu_p} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \left( \frac{N_A}{n_i^2 / N_D} \right)
\]

\[
V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}
\]

(Unit: Volts)
Built-In Potential Example

• Estimate the built-in potential for PN junction below.

  – Note that \( \frac{kT}{q} \ln(10) \approx 26\text{mV} \times 2.3 \approx 60\text{mV} \)

<table>
<thead>
<tr>
<th>N</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_D = 10^{17} \text{ cm}^{-3} )</td>
<td>( N_A = 10^{16} \text{ cm}^{-3} )</td>
</tr>
</tbody>
</table>

0.78 V

PN Junction under Reverse Bias

• A reverse bias increases the potential drop across the junction. As a result, the magnitude of the electric field increases and the width of the depletion region widens.

\[
W_{dep} = \sqrt{\frac{2\varepsilon \mu}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}
\]
Diode Current under Reverse Bias

- In equilibrium, the built-in potential effectively prevents carriers from diffusing across the junction.
- Under reverse bias, the potential drop across the junction increases; therefore, negligible diffusion current flows. A very small drift current flows, limited by the rate at which minority carriers diffuse from the quasi-neutral regions into the depletion region.

PN Junction Capacitance

- A reverse-biased PN junction can be viewed as a capacitor. The depletion width ($W_{\text{dep}}$) and hence the junction capacitance ($C_j$) varies with $V_R$.

$$C_j = \frac{\varepsilon_{\text{si}}}{W_{\text{dep}}}$$
Voltage-Dependent Capacitance

\[ C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} \]

\[ C_{j0} = \sqrt{\frac{\varepsilon_{si} q N_A N_D}{2 N_A + N_D V_0}} \]

\[ \varepsilon_{si} \approx 10^{-12} \text{ F/cm} \] is the permittivity of silicon.

More on the Built-In Potential \( (V_0) \)

Q: Why can’t we harness \( V_0 \) and use the PN junction as a battery?

A: A built-in potential also exists at a junction between a metal and a semiconductor (e.g. at a contact).

- If we connect the P and N regions together, there is no net voltage drop across the device:

\[ V_{bn} + V_0 + V_{bp} = 0 \]

No net current flows across the junction when the externally applied voltage is 0 V!
The Depletion Approximation

In the depletion region on the N side:

\[
\text{Gauss law } \quad \frac{dE}{dx} = \frac{\rho}{\varepsilon_{si}} = \frac{qN_D}{\varepsilon_{si}} \quad \varepsilon_{si} \approx 10^{-12} \text{F/cm}
\]

\[
E = \frac{qN_D}{\varepsilon_{si}} (x + b)
\]

In the depletion region on the P side:

\[
\frac{dE}{dx} = \frac{\rho}{\varepsilon_{si}} = -\frac{qN_A}{\varepsilon_{si}}
\]

\[
E = \frac{qN_A}{\varepsilon_{si}} (a - x)
\]

\[aN_A = bN_D\]

Effect of Applied Voltage

- The quasi-neutral N-type and P-type regions have low resistivity, whereas the depletion region has high resistivity.
  - Thus, when an external voltage \( V_D \) is applied across the diode, almost all of this voltage is dropped across the depletion region. (Think of a voltage divider circuit.)

- If \( V_D < 0 \) (reverse bias), the potential barrier to carrier diffusion is increased by the applied voltage.

- If \( V_D > 0 \) (forward bias), the potential barrier to carrier diffusion is reduced by the applied voltage.
**PN Junction under Forward Bias**

- A forward bias decreases the potential drop across the junction. As a result, the magnitude of the electric field decreases and the width of the depletion region narrows.

**Minority Carrier Injection under Forward Bias**

- The potential barrier to carrier diffusion is decreased by a forward bias; thus, carriers diffuse across the junction.
  - The carriers which diffuse across the junction become minority carriers in the quasi-neutral regions; they recombine with majority carriers, “dying out” with distance.

\[
\begin{align*}
\rho(x) &= qN_b \\
E(x) &= -qN_A \\
V(x) &= V_0 \\
n(x) &= n_n(x) \\
p(x) &= p_p(x)
\end{align*}
\]

**Equilibrium concentration of electrons on the P side:**

\[
n_{p0} = \frac{n^2}{N_A}
\]
Minority Carrier Concentrations at the Edges of the Depletion Region

- The minority-carrier concentrations at the edges of the depletion region are changed by the factor $e^{qV_d/kT} = e^{V_d/V_T}$
  - There is an excess concentration ($\Delta p_n$, $\Delta n_p$) of minority carriers in the quasi-neutral regions, under forward bias.

- Within the quasi-neutral regions, the excess minority-carrier concentrations decay exponentially with distance from the depletion region, to zero:

$$n_p(x') = n_{p0} + \Delta n_p(x')$$
$$\Delta n_p(x') = \frac{n_{p0}^2}{N_A} \left( e^{qV_d/V_T} - 1 \right) e^{-x'/L_n}$$

**Find the diffusion current density**

$$J_{n,diff} = \left| qD_n \frac{dn_p}{dx} \right| = \frac{qD_n n_{p0}^2}{N_A L_n} \left( e^{qV_d/V_T} - 1 \right) e^{-x'/L_n}$$

**Diode Current under Forward Bias**

- The current flowing across the junction is comprised of hole diffusion and electron diffusion components:

$$J_{tot} = J_{p,drift} \bigg|_{x=0} + J_{n,drift} \bigg|_{x=0} + J_{p,diff} \bigg|_{x=0} + J_{n,diff} \bigg|_{x=0}$$

- Assuming that the diffusion current components are constant within the depletion region (i.e. no recombination occurs in the depletion region):

$$J_{n,diff} \bigg|_{x=0} = \frac{qD_n n_{p0}^2}{N_A L_n} \left( e^{qV_d/V_T} - 1 \right)$$

$$J_{p,diff} \bigg|_{x=0} = \frac{qD_p n_{p0}^2}{N_D L_p} \left( e^{qV_d/V_T} - 1 \right)$$

$$J_{tot} = J_S \left( e^{qV_d/V_T} - 1 \right)$$

where $$J_S = qn_{p0}^2 \left( \frac{D_n}{N_A L_n} + \frac{D_p}{N_D L_p} \right)$$
Current Components under Forward Bias

- For a fixed bias voltage, $J_{\text{tot}}$ is constant throughout the diode, but $J_n(x)$ and $J_p(x)$ vary with position.

$I-V$ Characteristic of a PN Junction

- Current increases exponentially with applied forward bias voltage, and “saturates” at a relatively small negative current level for reverse bias voltages.

"Ideal diode" equation:

$$I_D = I_S \left( e^{\frac{V_D}{V_T}} - 1 \right)$$

$$I_S = AJ_S = Aq n_i^2 \left( \frac{D_n}{N_D L_n} + \frac{D_p}{N_P L_p} \right)$$
Parallel PN Junctions

- Since the current flowing across a PN junction is proportional to its cross-sectional area, two identical PN junctions connected in parallel act effectively as a single PN junction with twice the cross-sectional area, hence twice the current.

Diode Saturation Current $I_S$

$$I_S = A q n_i^2 \left( \frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

- $I_S$ can vary by orders of magnitude, depending on the diode area, semiconductor material, and net dopant concentrations.
  - typical range of values for Si PN diodes: $10^{-14}$ to $10^{-17}$ A/μm²
- In an asymmetrically doped PN junction, the term associated with the more heavily doped side is negligible:
  - If the P side is much more heavily doped, $I_S \approx A q n_i^2 \left( \frac{D_p}{L_p N_D} \right)$
  - If the N side is much more heavily doped, $I_S \approx A q n_i^2 \left( \frac{D_n}{L_n N_A} \right)$
What next

• PN Junction Contd...

Reverse Breakdown

• As the reverse bias voltage increases, the electric field in the depletion region increases. Eventually, it can become large enough to cause the junction to break down so that a large reverse current flows:
Reverse Breakdown Mechanisms

a) **Zener breakdown** occurs when the electric field is sufficiently high to pull an electron out of a covalent bond (to generate an electron-hole pair). \( E_{\text{zener}} \approx 10^6 \text{ V/m} \)

b) **Avalanche breakdown** occurs when electrons and holes gain sufficient kinetic energy (due to acceleration by the E-field) in-between scattering events to cause electron-hole pair generation upon colliding with the lattice.

![Diagram of Zener and Avalanche breakdown](image)

Summary

- Current flowing in a semiconductor is comprised of drift and diffusion components: \( J_{\text{tot}} = q\mu_e E + q\mu_h E + qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx} \)

- A region depleted of mobile charge exists at the junction between P-type and N-type materials.
  - A built-in potential drop \((V_0)\) across this region is established by the charge density profile; it opposes diffusion of carriers across the junction. A reverse bias voltage serves to enhance the potential drop across the depletion region, resulting in very little (drift) current flowing across the junction.
  - The width of the depletion region \((W_{\text{dep}})\) is a function of the bias voltage \((V_0)\).
    \[
    W_{\text{dep}} = \sqrt{\frac{2\varepsilon\mu_e (1 + \frac{1}{N_D}) (V_0 - V_\phi)}{q}} \\
    V_0 = \frac{kT}{q} \ln \frac{N_i N_D}{n_i^2}
    \]
Summary: PN-Junction Diode $I$-$V$

- Under forward bias, the potential barrier is reduced, so that carriers flow (by diffusion) across the junction
  - Current increases exponentially with increasing forward bias
  - The carriers become minority carriers once they cross the junction; as they diffuse in the quasi-neutral regions, they recombine with majority carriers (supplied by the metal contacts)
    "injection" of minority carriers
    $$ I_D = I_A \left( e^{V_A/V_T} - 1 \right) $$

- Under reverse bias, the potential barrier is increased, so that negligible carriers flow across the junction
  - If a minority carrier enters the depletion region (by thermal generation or diffusion from the quasi-neutral regions), it will be swept across the junction by the built-in electric field
    "collection" of minority carriers

Appendix

P-N Junction-Under Equilibrium

**Depletion approximation**

- Career depletion within space charge region
- Charge neutrality outside space charge

Dipole about the junction must have an equal number of charges on either side

$$ qAx_0 N_A = qAx_n N_D $$

Poisson's equation: Relates the gradient of the electric field to the local space charge at any point $x$

$$ \frac{d\varepsilon}{dx} = \frac{q}{\varepsilon} \left( p - n + N_D^+ - N_A^- \right) $$

$$ Q_+ = qAx_0 N_D $$

$$ Q_- = -qAx_0 N_D $$
Appendix

P-N Junction-Under Equilibrium

\[
\frac{d\epsilon}{dx} = \begin{cases} 
q_n - N_D - N_A^- & (0 < x < x_n) \\
q_n^+ - N_D^+ - N_A & (0 < x < x_{n0}) \\
q_n^0 - N_A & (-x_{p0} < x < 0) \\
\end{cases}
\]

\[
\int_{-x_{p0}}^{x_{n0}} d\epsilon = -\frac{q}{\epsilon_0} \int_{0}^{x_{p0}} dx
\]

\[
\epsilon_0 = -\frac{q}{\epsilon_0} N_A x_{p0} = -\frac{q}{\epsilon_0} N_D x_{n0}
\]

Appendix

P-N Junction-Under Equilibrium

\[
\epsilon(x) = -\frac{dV(x)}{dx} \quad \therefore V_{bi} = \int_{-x_{p0}}^{x_{n0}} \epsilon(x)dx
\]

\[
V_{bi} = -\frac{1}{2} W \epsilon_0 = \frac{1}{2} q N_A x_{p0} W = \frac{1}{2} q N_D x_{n0} W
\]

\[
x_{p0} N_A = x_{n0} N_D \therefore W = x_{p0} + x_{n0} \therefore x_{n0} = \frac{W N_A}{(N_A + N_D)}
\]

\[
V_{bi} = \frac{1}{2} q \frac{N_A N_D}{N_A + N_D} W^2
\]

\[
W = \left[ \frac{2}{q} V_{bi} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2}
\]
P-N Junction-Under Equilibrium

\[ W = \left[ \frac{2 e V_{bi}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2} \]

\[ x_{p0} = \frac{W N_D}{(N_A + N_D)} = \frac{2 e V_{bi}}{q} \left( \frac{N_D}{N_A(N_A + N_D)} \right)^{1/2} \]

The space charge/depletion region extends deep into the side with the lighter doping

\[ \varepsilon_0 = -\frac{q}{\varepsilon} N_A x_{p0} = -\frac{q}{\varepsilon} N_D x_{n0} \]

\[ V_{bi} = \frac{1}{2} \frac{q}{\varepsilon} \frac{N_A N_D}{(N_A + N_D)} W^2 \]

\[ W = \left[ \frac{2 e V_{bi}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2} \]

Constant-Voltage Diode Model

- If \( V_D < V_{D,\text{on}} \): The diode operates as an open circuit.
- If \( V_D \geq V_{D,\text{on}} \): The diode operates as a constant voltage source with value \( V_{D,\text{on}} \).
Example: Diode DC Bias Calculations

\[ V_X = I_X R_1 + V_D \approx I_X R_1 + V_T \ln \frac{I_X}{I_S} \]

- This example shows the simplicity provided by a constant-voltage model over an exponential model.
- Using an exponential model, iteration is needed to solve for current. Using a constant-voltage model, only linear equations need to be solved.

Small-Signal Analysis

- Small-signal analysis is performed at a DC bias point by perturbing the voltage by a small amount and observing the resulting linear current perturbation.
  - If two points on the \( I-V \) curve are very close, the curve in-between these points is well approximated by a straight line:

\[ \frac{\Delta I_D}{\Delta V_D} \approx \left. \frac{dI_D}{dV_D} \right|_{V_D=V_{D1}} \]

\[ = \frac{I_s}{V_T} e^{V_{D1}/V_T} \approx \frac{I_{D1}}{V_T} \]

\[ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \]
Diode Small-Signal Model

- Since there is a linear relationship between the small-signal current and small-signal voltage of a diode, the diode can be viewed as a linear resistor when only small changes in voltage are of interest.

\[
V_{D}(t) = V_0 + V_p \cos \omega t
\]

Small Sinusoidal Analysis

- If a sinusoidal voltage with small amplitude is applied in addition to a DC bias voltage, the current is also a sinusoid that varies about the DC bias current value.

\[
I_D(t) = I_0 + I_p \cos \omega t \approx I_s \exp \left( \frac{V_0}{V_T} \right) + \frac{V_p \cos \omega t}{V_T / I_0}
\]
Cause and Effect

• In (a), voltage is the cause and current is the effect. In (b), current is the cause and voltage is the effect.

\[ \Delta V_D = I_D \Delta v \]

\[ \Delta I_D = \frac{\Delta V_D}{r_d} = \frac{I_D L_D}{V_T} \]

\[ \Delta V_D = \Delta I_D \frac{r_d}{I_D} \]

What next

• PN Junction Contd..., BJT...