Complexity Theory
Problem Set 4

1. Say that a class C_1 is superior to a class C_2 if there is a machine M_1 in class C_1 such that for every machine M_2 in class C_2 and every large enough n, there is an input of size between n and n^2 on which M_1 and M_2 answer differently. Is DTIME($n^{1.1}$) superior to DTIME (n)?

2. Prove or disprove. If P $\neq$ NP, $\exists\, L_1, L_2 \in$ NP, such that L_1, L_2 are not empty set or Σ^* and neither $L_1 \leq_p L_2$ nor $L_2 \leq_p L_1$. (Warning: I don't have an answer for this problem. It might be too hard.)

3. Show that if *3SAT* is polynomial-time reducible to *3SAT*, then PH $=$ NP.

4. Try to modify some **NP-complete** problems so that they remain in Σ_2^p but not seemingly in **NP**.

5. The class **DP** is defined as the set of languages L for which there are two languages $L_1 \in$ NP, $L_2 \in$ **coNP**, such that $L = L_1 \cap L_2$. (Do not confuse **DP** with **NP** ∩ **coNP**, which may seem superficially similar.) Show that

(a) *EXACT-INDSET* $\in \Pi_2^p$

(b) *EXACT-INDSET* $\in$ DP.

(c) Every language in **DP** is polynomial-time reducible to *EXACT-INDSET*.

6. Suppose A is some language such that P$^A =$ NP^A. Then, show that PH$^A \subseteq$ P^A.

Solutions

1. *https://cs.stackexchange.com/questions/54801/problem-in-computational-complexity-superior-class*

2. I do not have the answer.

3. If *3SAT* is polynomial-time reducible to $\overline{\textit{3SAT}}$, then **NP** $\subseteq$ **coNP**.
NP $\subseteq$ **coNP** $\Longrightarrow$ **NP** $=$ **coNP**. Thus hierarchy collapses to **NP**.

4. You can convert problems like *VertexCover*, etc.

5. *https://zoo.cs.yale.edu/classes/cs468/previous-years/spr15/solutions/HW3-Solutions.pdf*

6. Go through the proof of **P** $=$ **NP** implies **PH** $=$ **NP** and see whether the result holds true for and oracle A as well.