ADSA
Problem Set 2

1. Given an element x in an n-node augmented tree (the one which can find the rank of an element or find the element of the given rank) and a natural number i, show how to determine the ith successor of x in $O(\lg n)$ time.

2. Show how to use an augmented tree to count the number of inversions (a pair of elements $A[i]$ and $A[j]$ is an inversion if $A[i] > A[j]$ where $i < j$) in an array A of n distinct elements in time $O(n \lg n)$.

3. In the interval tree, show how you can maintain max values at every node during rotations in $O(1)$ time.

4. Prove that a sequence of m **Make-Set**, **Union**, and **Find-Set** operations, first n of which are **Make-Set** operations, takes $O(m + n \log n)$ time under the above heuristic of appending the shorter list at the end of the longer list.

5. Give a sequence of m **Make-Set**, **Union**, and **Find-Set** operations, first n of which are **Make-Set** operations, that takes $\Omega(m \log n)$ time when using **Union** by rank not path compression.

Solutions

Solution 1

See 14.1-5 here.

Solution 2

See 14.1-7 here.

Solution 3

See 14.3-1 here.

Solution 4

Read Theorem 19.1 in CLRS (4th edition).

Solution 5

See 21.3-3 here.